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A NOTE ON LEVI'S PROBLEM WITH DISCONTINUOUS FUNCTIONS

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§ 1. INTRODUCTION

In [3] Fornaess and Narasimhan proved that a complex space X which carries a strongly plurisubharmonic exhaustion function $\varphi: X \to \mathbf{R}$ is a Stein space. It is a remarkable fact that φ is supposed only upper semicontinuous.

A natural question which arises when we consider the Levi problem with upper semicontinuous functions is the following: what would happen if we allowed φ to take on the value $-\infty$. Simple examples (compact complex spaces, the blowing up of $\mathbf{C}^n$ at the origine...) show us that X is not necessarily Stein. The best result one might hope to obtain is X being 1-convex.

The aim of this short note is to give an affirmative answer to this question, hence to prove the following theorem conjectured by Fornaess and Narasimhan:

THEOREM 1. *Let X be a complex space which admits a strongly plurisubharmonic exhaustion function $\varphi: X \to [-\infty, \infty)$. Then X is 1-convex.*

If φ is supposed real-valued it follows easily, from the maximum principle, that the exceptional set of X is empty, hence X is Stein. This is exactly Fornaess-Narasimhan's theorem.

§ 2. PRELIMINARIES

All complex spaces are assumed to be reduced and countable at infinity.

An upper semicontinuous function $\varphi: X \to [-\infty, \infty)$ is called plurisubharmonic if for every holomorphic map $\tau: W \to X$ (W = the unit disc in $\mathbf{C}$) it follows that $\varphi \circ \tau$ is subharmonic on W (possibly $\equiv -\infty$). φ is said