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# A NOTE ON LEVI'S PROBLEM WITH DISCONTINUOUS FUNCTIONS

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## § 1. INTRODUCTION

In [3] Fornaess and Narasimhan proved that a complex space $X$ which carries a strongly plurisubharmonic exhaustion function $\varphi: X \to \mathbf{R}$ is a Stein space. It is a remarkable fact that $\varphi$ is supposed only upper semicontinuous.

A natural question which arises when we consider the Levi problem with upper semicontinuous functions is the following: what would happen if we allowed $\varphi$ to take on the value $-\infty$. Simple examples (compact complex spaces, the blowing up of $\mathbf{C}^n$ at the origine...) show us that $X$ is not necessarily Stein. The best result one might hope to obtain is $X$ being 1-convex.

The aim of this short note is to give an affirmative answer to this question, hence to prove the following theorem conjectured by Fornaess and Narasimhan:

THEOREM 1. *Let $X$ be a complex space which admits a strongly plurisubharmonic exhaustion function $\varphi: X \to [-\infty, \infty)$. Then $X$ is 1-convex.*

If $\varphi$ is supposed real-valued it follows easily, from the maximum principle, that the exceptional set of $X$ is empty, hence $X$ is Stein. This is exactly Fornaess-Narasimhan's theorem.

## § 2. PRELIMINARIES

All complex spaces are assumed to be reduced and countable at infinity.

An upper semicontinuous function $\varphi: X \to [-\infty, \infty)$ is called plurisubharmonic if for every holomorphic map $\tau: W \to X$ ($W$ = the unit disc in $\mathbf{C}$) it follows that $\varphi \circ \tau$ is subharmonic on $W$ (possibly $\equiv -\infty$). $\varphi$ is said