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to make (*) correct. We note that this is equivalent to choosing C_l for $L_n + 1 \leq l \leq L_{n+1}$. This completes the construction.

To verify that (Q_k) is dense, let E be an entire function, $N > 0$ and $\varepsilon > 0$. Choose n so that

$$\text{i) } n > N,$$

$$\text{ii) } |h_n(z) - E(z)| \leq \frac{1}{3} \varepsilon \quad \text{for } |z| \leq N,$$

$$\text{iii) } \frac{1}{n} \leq \frac{1}{3} \varepsilon, \quad \text{and}$$

$$\text{iv) } |(I^{L_n} f)(z)| \leq \frac{1}{3} \varepsilon \quad \text{for } |z| \leq N.$$

Then $|Q_{L_n}(z) - E(z)| \leq \varepsilon$ for all $|z| \leq N$.

We conclude with a result communicated to us by I. N. Baker, namely that there is no entire function f such that the sequence of its compositional iterates $f^{[n]}$ spans a dense set. If f is linear, then the closed span of its iterates contains only linear functions. (That is, the only univalent entire functions are the linear ones. See [BUR], §11.19, p. 370 and the Notes on p. 407 for several proofs.) If f is not linear, then there exist z_1, z_2 with $z_1 \neq z_2$ but $f(z_1) = f(z_2)$. This equality also holds for all linear combinations of iterates, so the closed span of the iterates lies in the proper closed subspace $\{g : g(z_1) = g(z_2)\}$. Finally, we remark that if f is universal in sense (b) then the pair $f(z), z - 1$ is universal under composition.

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