

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 30 (1984)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: FOUR CHARACTERIZATIONS OF REAL RATIONAL DOUBLE POINTS
Autor: Durfee, Alan H.

Bibliographie
DOI: <https://doi.org/10.5169/seals-53818>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 27.04.2026

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

Finally, it remains to verify that the equations $f(x, y, z) = 0$ in column (a') have the resolutions of column (b'). This can be verified by blowing up points in three-space. (Note that by $(4B \Leftrightarrow 4A)$ above, the graphs are already known, and it is only necessary to match them with the equations.) The details are hard to write down, and will be omitted. This completes the proof of Theorem 2.

It would be interesting to understand the resolution of real singularities better. For example, what does the resolution of $x^3 + y^2 - z^2 = 0$ look like, topologically? It would also be interesting to understand the connections (if any) between the modality of a real germ as real germ and as complex germ.

More generally, the Dynkin diagrams B_k , C_k , and F_4 arise in situations where there is an involution on an object corresponding to the diagrams A_k , D_k and E_k . In the above theorem, the involution is conjugation. Connections with simple algebraic groups are discussed in [Slodowy, 6.2 and Appendix I]. Another example is critical points of functions on manifolds with boundary [Arnold 2]; these correspond to functions on the doubled manifold invariant with respect to the obvious involution. Lastly, the diagram G_2 arises where there is an automorphism of order 3 on an object corresponding to D_4 .

BIBLIOGRAPHY

- ARNOLD, V. [1] Normal forms of functions near degenerate critical points, The Weyl groups A_k , D_k , E_k , and Lagrange singularities. *Funk. Anal.* 6 (1972), 3-25.
- [2] Critical points of functions on a manifold with boundary, the simple Lie groups B_k , C_k , and F_4 and singularities of evolutes. *Russian Math. Surveys* 33 (5) (1978), 99-116.
- DURFEE, A. Fifteen characterizations of rational double points and simple critical points. *L'Enseignement Math.* 25 (1979), 131-163.
- LIPMAN, J. Rational singularities, with applications to algebraic surfaces and unique factorization. *Publ. Math. de l'Inst. des Hautes Etudes Scientifiques* 36 (1969), 195-279.
- SIERSMA, D. *Classification and deformation of singularities*. Thesis, University of Amsterdam, 1974.
- SLODOWY, P. *Simple singularities and simple algebraic groups*. Lecture Notes in Math. No. 815, Springer-Verlag, 1980.
- WAHL, J. Derivations, automorphisms, and deformations of quasi-homogeneous singularities. *Proc. Symp. Pure Math.* 40 (1983), 613-624, American Math. Society.

(Reçu le 5 août 1982)

Alan H. Durfee

Smith College
Northampton, MA 01063