

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 29 (1983)  
**Heft:** 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** SOME KNOT THEORY OF COMPLEX PLANE CURVES  
**Autor:** Rudolph, Lee  
**Kapitel:** §3. RÉSUMÉ OF BASIC DEFINITIONS  
**DOI:** <https://doi.org/10.5169/seals-52979>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 18.04.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

interior of a ball or a bidisk), well-behaved at the boundary; a knot-theorist can study either of two codimension-2 situations—the complex curve in its ambient space, or the boundary of this pair.

This middle panel of the triptych has been less studied than the other two, though it is of obvious relevance to both.

### §3. RÉSUMÉ OF BASIC DEFINITIONS

By *complex surface* I mean a smooth manifold of 4 real dimensions, equipped with a complex structure. A *complex curve*  $\Gamma$  in a complex surface  $M$  is a closed subset which is locally of the form  $\{(z, w) \in U \subset \mathbf{C}^2 : f(z, w) = 0\}$  where  $f : U \rightarrow \mathbf{C}$  is a nonconstant complex analytic function. A *Riemann surface* is a smooth manifold of 2 real dimensions, equipped with a complex structure.

It is a fundamental fact, to which is due the especial appositeness of classical knot theory to the study of curves in surfaces, that any complex curve  $\Gamma \subset M$  has a *resolution* of the following sort: There is a Riemann surface  $R$ , and a holomorphic mapping  $r : R \rightarrow M$ , so that  $r(R) = \Gamma$ ; in fact, there is a discrete (possibly empty) subset  $\mathcal{S}(\Gamma) \subset \Gamma$ , the *singular locus of  $\Gamma$  in  $M$* , so that the *regular locus*  $\mathcal{R}(\Gamma) = \Gamma - \mathcal{S}(\Gamma)$  is a Riemann surface, and  $R$  is the union (with what turns out to be a unique topology and complex structure) of  $\mathcal{R}(\Gamma)$ , on which  $r$  is the identity, and a discrete set  $r^{-1}(\mathcal{S}(\Gamma)) \subset R$  mapping finitely-to-one onto  $\mathcal{S}(\Gamma)$ .

The singular locus is, of course, exactly the set of points of  $\Gamma$  at which, no matter what the local representation of  $\Gamma$  as the zeroes of an analytic function  $f(z, w)$ , the (complex) gradient vector  $\nabla f$  vanishes.

If  $P$  is a point of  $\Gamma$ , and  $Q \in r^{-1}(P) \subset R$ , then the germ at  $P$  of the  $r$ -image of a small disk on  $R$  centered at  $Q$  is called a *branch* of  $\Gamma$  at  $P$ . (Abusively, “branch” may also be used below to refer to some representatives of this germ.) Naturally, at a regular point there is only one branch; but there may be only one branch at a point, and the point still be singular.

References: [G-R], [Mi 2].

### §4. LOCAL KNOT THEORY IN BRIEF

Using local coordinates in the resolution  $R$  and the ambient surface  $M$ , one sees that each branch of a curve  $\Gamma$  can be parametrized either by  $z = t, w = 0$  or (more interestingly) by some pair  $z = t^m, w = t^n + c_{n+1}t^{n+1} + \dots$