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Autor: Rudolph, Lee

Kapitel: §3. RÉSUMÉ OF BASIC DEFINITIONS

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interior of a ball or a bidisk), well-behaved at the boundary; a knot-theorist can study either of two codimension-2 situations—the complex curve in its ambient space, or the boundary of this pair.

This middle panel of the triptych has been less studied than the other two, though it is of obvious relevance to both.

§3. Résumé of basic definitions

By *complex surface* I mean a smooth manifold of 4 real dimensions, equipped with a complex structure. A *complex curve* Γ in a complex surface M is a closed subset which is locally of the form $\{(z, w) \in U \subset \mathbf{C}^2 : f(z, w) = 0\}$ where $f : U \to \mathbf{C}$ is a nonconstant complex analytic function. A *Riemann surface* is a smooth manifold of 2 real dimensions, equipped with a complex structure.

It is a fundamental fact, to which is due the especial appositeness of classical knot theory to the study of curves in surfaces, that any complex curve $\Gamma \subset M$ has a *resolution* of the following sort: There is a Riemann surface R, and a holomorphic mapping $r : R \to M$, so that $r(R) = \Gamma$; in fact, there is a discrete (possibly empty) subset $\mathscr{S}(\Gamma) \subset \Gamma$, the *singular locus of* Γ *in* M, so that the *regular locus* $\mathscr{R}(\Gamma) = \Gamma - \mathscr{S}(\Gamma)$ is a Riemann surface, and R is the union (with what turns out to be a unique topology and complex structure) of $\mathscr{R}(\Gamma)$, on which r is the identity, and a discrete set $r^{-1}(\mathscr{S}(\Gamma)) \subset R$ mapping finitely-to-one onto $\mathscr{S}(\Gamma)$.

The singular locus is, of course, exactly the set of points of Γ at which, no matter what the local representation of Γ as the zeroes of an analytic function $f(z, w)$, the (complex) gradient vector ∇f vanishes.

If P is a point of Γ, and $Q \in r^{-1}(P) \subset R$, then the germ at P of the r-image of a small disk on R centered at Q is called a *branch* of Γ at P. (Abusively, "branch" may also be used below to refer to some representatives of this germ.) Naturally, at a regular point there is only one branch; but there may be only one branch at a point, and the point still be singular.

References: [G-R], [Mi 2].

§4. Local knot theory in brief

Using local coordinates in the resolution R and the ambient surface M, one sees that each branch of a curve Γ can be parametrized either by $z = t, w = 0$ or (more interestingly) by some pair $z = t^m$, $w = t^n + c_{n+1}t^{n+1} + \ldots$