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# SOME KNOT THEORY OF COMPLEX PLANE CURVES [1])

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## §1. ASPECTS OF THE "PLACEMENT PROBLEM" FOR COMPLEX PLANE CURVES

How can a complex curve be placed in a complex surface?

The question is vague; many different ways to make it more specific may be imagined. The theory of deformations of complex structure, and their associated moduli spaces, is one way. Differential geometry and function theory, curvatures and currents, could be brought in. Even the generalized Nevanlinna theory of value distribution, for analytic curves, can somehow be construed as an aspect of the "placement problem".

By "knot theory" I mean to connote those aspects of the situation that are more immediately topological. I hope to show that there is something of interest there.

## §2. A TRIPTYCH

Here are three ways to interpret the phrase "knot theory of complex plane curves".

Globally: the "complex plane" is projective space $\mathbf{CP}^2$ or affine space $\mathbf{C}^2$; a "curve" is an algebraic curve (in projective space) or an algebraic or analytic curve (in affine space); here, "knot theory" has historically been largely concerned with studying the "knot group", though there are also results on "knot type".

Locally: a "complex plane curve" is the germ of a plane curve (algebraic, analytic, or formal) over $\mathbf{C}$; this is the study of singularities, and "knot theory" has been the classical knot theory of links in the 3-sphere, put to work in the service of that study.

In between: a "complex plane curve" is an analytic curve in a reasonable open set in a complex surface (chiefly, in the theory as so far developed, the

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