

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 29 (1983)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ON FREE SUBGROUPS OF SEMI-SIMPLE GROUPS
Autor: Borel, A.
Kapitel: §3. Compact groups. Proof of Theorem A.
DOI: <https://doi.org/10.5169/seals-52977>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 19.08.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

§3. COMPACT GROUPS. PROOF OF THEOREM A.

1. Let U be a compact Lie group. Then we may view U as the group $G(\mathbf{R})$ of real points of an algebraic group G defined over \mathbf{R} [5]. Furthermore, the maximal (topological) tori of U are the groups $T(\mathbf{R})$, where T runs through the maximal \mathbf{R} -tori of G . They are conjugate under inner automorphisms of U . Corollary 1 to Theorem 2 insures the existence of a non-commutative free subgroup Γ of U such that every $\gamma \in \Gamma - \{1\}$ is strongly regular, i.e., generates a dense subgroup of a maximal torus of U . If now V is a closed subgroup of U , then, by [10], $\chi(U/V) = 0$ if V does not contain a maximal torus of U , and is equal to $[N_U(T) : N_V(T)]$ if V contains a maximal torus T of U . By the results just recalled, we may write $V = H(\mathbf{R})$, where H is an algebraic \mathbf{R} -subgroup of G , the condition (*) of §2 is satisfied, and any maximal torus of U is conjugate to T . Theorem A now follows from Corollaries 1 and 3 to Theorem 2.

2. The results of this paper, specialized to compact Lie groups, can of course be proved more directly, in the framework of the theory of compact Lie groups, without recourse to the theory of algebraic groups. For the benefit of the reader mainly interested in that case, we sketch how to modify the above arguments.

The main point is again to prove Theorem 1, where now G stands for a non-trivial compact connected semi-simple Lie group. In part a) of the proof, the role of \mathbf{SL}_n is taken by \mathbf{SU}_n . If $n = 2$, G contains non-commutative free subgroups. If $n > 2$, the argument is the same except that now we take for D , exactly as in [8], a division algebra with an involution of the second kind and identify \mathbf{SU}_n to $(D \otimes_L \mathbf{R})^1$, where L is the fixed field, in the center of D , of the given involution of D . In part b), we use the fact that if G is simple, not locally isomorphic to \mathbf{SU}_n , then it contains a proper closed connected semi-simple subgroup of maximal rank, for which we can refer directly to [2] (the proof of Lemma 1 was in fact just an adaptation to algebraic groups of the one in [2]).

Then, as pointed out in section 5 of §2, a simple category argument yields Theorem 2, whence also Corollary 1 to Theorem 2 and Theorem A.

§4. FREE GROUP ACTIONS WITH COMMUTATIVE ISOTROPY GROUPS

1. Let Γ be a non-commutative free group acting on a set X . Assume that Γ acts freely, or more generally, that the isotropy groups $\Gamma_x (x \in X)$ are commutative (hence cyclic), and that at least one is reduced to $\{1\}$. Then the decomposition theorem 2.2.1, 2.2.2 of [6] implies in particular the following: given $n \geq 2$, there exists a partition of X into $2n$ subsets X_i and elements $\gamma_i \in \Gamma (1 \leq i \leq 2n)$ such that X is the disjoint union of $\gamma_i X_i$ and $\gamma_{n+i} X_{n+i} (i \leq i \leq n)$. If we view the operations of