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This shows that D has involutions of the second kind if and only if the class of D_1 in $\text{Br}(k)$ is trivial. This class is the required norm $N_{K/k}(\text{cl}(D))$. In the localization spirit, this can be deduced from the fact that the homothety by $\lambda \in K'^*$ of V' induces on W' the homothety by $N_{K'/k'}(\lambda) \in k'^*$.

APPENDIX C

For $n \geq 3$, examples can be obtained as follows: take $k' = \mathbf{Q}[\zeta]$, with $\zeta = \exp(2\pi i/n)$, and $k = k' \cap \mathbf{R}$. Fix $a, b \in k^*$ and let D be the k' -algebra generated by X, Y , subject to

$$\begin{aligned} X^n &= a, & Y^n &= b \\ XY &= \zeta YX. \end{aligned}$$

It admits the anti-involution $*$, inducing complex conjugation on k' , defined by $\zeta^* = \zeta^{-1}$, $X^* = X$, $Y^* = Y$. The algebra D is of the type we require, provided it is a division algebra. This happens already with $a, b \in \mathbf{Z}$: take for a a prime congruent to 1 mod n , and for b an integer whose residue mod a has in the cyclic group of order n $(\mathbf{Z}/(a))^*/(\mathbf{Z}/(a))^{*n}$ an image of exact order n . For instance $n = 3$, $a = 7$, $b = 2$. For $n = 2$, one proceeds similarly with $k' = \mathbf{Q}[i]$, $\zeta = -1$, a congruent to 1 mod 4 and b not a square mod a . For instance, $a = 5$, and $b = 2$. In each case, the assumption on a ensures that k' embed in the a -adic completion \mathbf{Q}_a of \mathbf{Q} , and the fact that D is a division algebra can be seen locally at a : $D \otimes_{k'} \mathbf{Q}_a$ is a division algebra with center \mathbf{Q}_a .