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Autor: de la Harpe, Pierre
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homomorphism $PGL(n, \mathbf{C}) \to PGL(N, \mathbf{C})$. Then $\lambda_k(\tilde{\gamma})$ has eigenvalues $\nu_1, ..., \nu_N$ with $|\nu_1| > |\nu_j|$ for $j = 2, ..., N$. By lemma 3, there exists a $\lambda_k(\tilde{\Gamma})$-irreducible subspace W_0 of $\mathbf{C}^N$, associated to a representation $\sigma_0 : \tilde{\Gamma} \to GL(W_0)$, such that ν_1 is an eigenvalue of $\sigma_0(\tilde{\gamma})$. As the Z-closure $\tilde{G}$ of $\tilde{\Gamma}$ in $SL(n, \mathbf{C})$ is semi-simple, the group $\tilde{G}$ is perfect and $\sigma_0(\tilde{\Gamma})$ lies in $SL(W_0)$. As $|\nu_1| > 1$, one has $\dim_{\mathbf{C}} W_0 \geqslant 2$.

Thus one may assume from the start that Γ contains a sharp semi-simple element, and indeed by lemmas 1 and 2 two very sharp elements in general position. The conclusion follows as in case 2 of the proof of the proposition in section 4. □

Now lemma 1 remains true without the hypothesis "semi-simple". This has been announced by Y. Guivarch', who uses ideas of H. Fürstenberg to show the following: given an appropriate subset S of Γ containing a sharp element, then almost any "long" word in the letters of S is very sharp. Using this, one may replace (ii) in the theorem above by the following a priori weaker hypothesis

(ii′) Γ is not relatively compact.

Then, one first checks as for theorem 2 of section 4 that Γ contains hyperbolic elements; one concludes as in the previous proof, with Guivarch's version of lemma 1.

For subgroups of $PU(n)$, one may repeat the discussion at the end of section 4.

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Pierre de la Harpe
Section de mathématiques
C.P. 240
CH-1211 Genève 24