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3. THE THEORY OF THE HIGHEST WEIGHT

Before decomposing the \mathfrak{sl}_2 -space \mathcal{A} we must review the finite dimensional representation theory of \mathfrak{sl}_2 .

The *weight vectors* of an \mathfrak{sl}_2 -representation W are the eigenvectors of H in W . The *weights* of W are the eigenvalues of its nonzero weight vectors.

Every finite dimensional \mathfrak{sl}_2 -module is spanned by its weight vectors. The weights of such a representation are all integers and are thus ordered by the usual order on \mathbf{R} . The largest of a finite set of integral weights is traditionally referred to as the *highest weight*.

Two finite dimensional irreducible \mathfrak{sl}_2 -representations are isomorphic if and only if they have the same highest weights, which are necessarily nonnegative.

The element $X^a Y^b$ of V is a weight vector of weight $a-b$. This shows that X^m is a vector of highest weight m in V_m and therefore that the V_m for $m \geq 0$ form a set of representatives of the equivalence classes of finite dimensional irreducible \mathfrak{sl}_2 -representations; which is precisely why we are studying them in this paper.

The last general fact which we will recall without proof is this: every finite dimensional representation of \mathfrak{sl}_2 is a direct sum of irreducible representations.

Given a representation W of \mathfrak{sl}_2 which is a sum of finite dimensional representations one often wishes to write it explicitly as a direct sum of irreducible representations, that is, of representations isomorphic to the V_m . A method for doing this is provided by the observation that the space of weight vectors of highest weight in V_m is the space annihilated by E_+ and is one dimensional. Thus for each $v \in W$ of weight m such that $E_+ v = 0$, there is a unique \mathfrak{sl}_2 -homomorphism from V_m to W taking X^m to v . The explicit decomposition of W therefore amounts to the determination of a basis consisting of weight vectors of the kernel of E_+ in W .

4. THE DECOMPOSITION OF \mathcal{A}

We apply the procedure of the last paragraph to the representation of \mathfrak{sl}_2 on \mathcal{A} . By definition of ρ the kernel of $\rho(E_+)$ is just the commutant of E_+ in \mathcal{A} .

Let \mathcal{B} be the subalgebra of \mathcal{A} generated by X , ∂_Y , and J .