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### 3. THE THEORY OF THE HIGHEST WEIGHT

Before decomposing the  $\mathfrak{sl}_2$ -space  $\mathcal{A}$  we must review the finite dimensional representation theory of  $\mathfrak{sl}_2$ .

The *weight vectors* of an  $\mathfrak{sl}_2$ -representation  $W$  are the eigenvectors of  $H$  in  $W$ . The *weights* of  $W$  are the eigenvalues of its nonzero weight vectors.

Every finite dimensional  $\mathfrak{sl}_2$ -module is spanned by its weight vectors. The weights of such a representation are all integers and are thus ordered by the usual order on  $\mathbf{R}$ . The largest of a finite set of integral weights is traditionally referred to as the *highest weight*.

Two finite dimensional irreducible  $\mathfrak{sl}_2$ -representations are isomorphic if and only if they have the same highest weights, which are necessarily nonnegative.

The element  $X^a Y^b$  of  $V$  is a weight vector of weight  $a-b$ . This shows that  $X^m$  is a vector of highest weight  $m$  in  $V_m$  and therefore that the  $V_m$  for  $m \geq 0$  form a set of representatives of the equivalence classes of finite dimensional irreducible  $\mathfrak{sl}_2$ -representations; which is precisely why we are studying them in this paper.

The last general fact which we will recall without proof is this: every finite dimensional representation of  $\mathfrak{sl}_2$  is a direct sum of irreducible representations.

Given a representation  $W$  of  $\mathfrak{sl}_2$  which is a sum of finite dimensional representations one often wishes to write it explicitly as a direct sum of irreducible representations, that is, of representations isomorphic to the  $V_m$ . A method for doing this is provided by the observation that the space of weight vectors of highest weight in  $V_m$  is the space annihilated by  $E_+$  and is one dimensional. Thus for each  $v \in W$  of weight  $m$  such that  $E_+ v = 0$ , there is a unique  $\mathfrak{sl}_2$ -homomorphism from  $V_m$  to  $W$  taking  $X^m$  to  $v$ . The explicit decomposition of  $W$  therefore amounts to the determination of a basis consisting of weight vectors of the kernel of  $E_+$  in  $W$ .

### 4. THE DECOMPOSITION OF $\mathcal{A}$

We apply the procedure of the last paragraph to the representation of  $\mathfrak{sl}_2$  on  $\mathcal{A}$ . By definition of  $\rho$  the kernel of  $\rho(E_+)$  is just the commutant of  $E_+$  in  $\mathcal{A}$ .

Let  $\mathcal{B}$  be the subalgebra of  $\mathcal{A}$  generated by  $X$ ,  $\partial_Y$ , and  $J$ .