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Dupont and Sah show that the volume function and the sharpened Dehn invariant can be incorporated into a single function  $\rho$ , as follows. Let

$$\rho(z) = 1 \wedge L(z) - 1 \wedge L(1-z) + l(z) \wedge l(1-z),$$

with values in  $\wedge^2\mathbb{C}$ , where  $l(z) = \log(z)/2\pi i$  and

$$L(z) = \mathcal{L}_2(z)/4\pi^2 = \int_0^1 l(1-z)dl(z).$$

This expression is certainly well defined in the strip  $0 < \operatorname{Re}(z) < 1$ , and satisfies  $\rho(z) + \rho(1-z) = 0$ . If we analytically continue each of its constituent functions in a loop around zero or one, then the expression  $\rho(z)$  remains unchanged. Hence  $\rho$  is well defined as a mapping from  $\mathbb{C} - \{0, 1\}$  to  $\wedge^2\mathbb{C}$ . They show that  $\rho$  also satisfies the symmetry condition (31), the Kubert identity (32), and the cocycle equation (33).

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