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Artikel: ON POLYLOGARITHMS, HURWITZ ZETA FUNCTIONS, AND THE KUBERT IDENTITIES

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Dupont and Sah show that the volume function and the sharpened Dehn invariant can be incorporated into a single function ρ, as follows. Let

$$\rho(z) = 1 \wedge L(z) - 1 \wedge L(1-z) + l(z) \wedge l(1-z),$$

with values in $\wedge^2 \mathbf{C}$, where $l(z) = \log(z)/2\pi i$ and

$$L(z) = \mathscr{L}_2(z)/4\pi^2 = \int_0 l(1-z)dl(z).$$

This expression is certainly well defined in the strip $0 < \operatorname{Re}(z) < 1$, and satisfies $\rho(z) + \rho(1-z) = 0$. If we analytically continue each of its constituent functions in a loop around zero or one, then the expression $\rho(z)$ remains unchanged. Hence ρ is well defined as a mapping from $\mathbf{C} - \{0, 1\}$ to $\wedge^2 \mathbf{C}$. They show that ρ also satisfies the symmetry condition (31), the Kubert identity (32), and the cocycle equation (33).

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