

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 28 (1982)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: REDUCIBILITY BY ALGEBRAIC PROJECTIONS
Autor: Valiant, L. G.
Kurzfassung: Abstract
DOI: <https://doi.org/10.5169/seals-52240>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 10.08.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

REDUCIBILITY BY ALGEBRAIC PROJECTIONS *

by L. G. VALIANT

ABSTRACT

Substitution as a notion of reduction between two polynomials or two Boolean functions is considered. It is shown that in a strong sense linear programming is a universal technique for computing discrete functions in polynomial time. The robustness of the notion of p -definability for polynomials is demonstrated by showing that alternative formulations, whether based on formula or program size, are equivalent. Also it is closed under most natural operations including substitution, taking coefficients and differentiation. These results facilitate the recognition of particular polynomials as p -definable. The polynomial analogue of the Meyer-Stockmeyer hierarchy collapses.

1. INTRODUCTION

The programming concept of a *subroutine* is well represented in theoretical computer science in the notion of *reducibility*. A function $A(\mathbf{x})$ is many-one reducible to function $B(\mathbf{y})$ if there is an easily computed transformation f such that $A(\mathbf{x}) = B(f(\mathbf{x}))$. A can be computed by computing f and then calling a subroutine for B . Traditionally this is the strictest notion considered. It is relaxed sometimes to allow several subroutine calls, or further computation after the call. In this paper we proceed in the opposite direction by considering reductions stricter still.

We say that $A(x_1, \dots, x_n)$ is a *projection* of $B(y_1, \dots, y_m)$ if after substituting for each y_i either an x_j or a constant, B equals $A(x_1, \dots, x_n)$. Mathematically this notion has the obvious advantages of simplicity and of independence from any computational models. In programming terms it corresponds naturally to the concept of a *package* rather than subroutine,

* This article has already been published in *Logic and Algorithmic*, an international Symposium in honour of Ernst Specker, Zürich, February 1980. Monographie de L'Enseignement Mathématique N° 30, Genève 1982.