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REDUCIBILITY BY ALGEBRAIC PROJECTIONS *

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ABSTRACT

Substitution as a notion of reduction between two polynomials or two Boolean functions is considered. It is shown that in a strong sense linear programming is a universal technique for computing discrete functions in polynomial time. The robustness of the notion of p-definability for polynomials is demonstrated by showing that alternative formulations, whether based on formula or program size, are equivalent. Also it is closed under most natural operations including substitution, taking coefficients and differentiation. These results facilitate the recognition of particular polynomials as p-definable. The polynomial analogue of the Meyer-Stockmeyer hierarchy collapses.

1. INTRODUCTION

The programming concept of a *subroutine* is well represented in theoretical computer science in the notion of *reducibility*. A function $A(\mathbf{x})$ is many-one reducible to function $B(\mathbf{y})$ if there is an easily computed transformation f such that $A(\mathbf{x}) = B(f(\mathbf{x}))$. A can be computed by computing f and then calling a subroutine for B. Traditionally this is the strictest notion considered. It is relaxed sometimes to allow several subroutine calls, or further computation after the call. In this paper we proceed in the opposite direction by considering reductions stricter still.

We say that $A(x_1, ..., x_n)$ is a *projection* of $B(y_1, ..., y_m)$ if after substituting for each y_i either an x_j or a constant, B equals $A(x_1, ..., x_n)$. Mathematically this notion has the obvious advantages of simplicity and of independence from any computational models. In programming terms it corresponds naturally to the concept of a *package* rather than subroutine,

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