Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 28 (1982)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE REPRESENTATION THEORY OF SL(2, R), A NON-INFINITESIMAL APPROACH

Autor: Koornwinder, Tom H.

Kapitel: 2.1. Preliminaries

DOI: https://doi.org/10.5169/seals-52233

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Mehr erfahren

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. En savoir plus

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. Find out more

Download PDF: 10.01.2026

ETH-Bibliothek Zürich, E-Periodica, https://www.e-periodica.ch

$SU(n, 1)$. For convenience, in order to avoid matrix manipulations, we restrict ourselves here to the case that the compact subgroup K is abelian.

The results of this paper may be generalized rather easily to the universal covering group of $SL(2, \mathbf{R})$. The extension to $SL(2, \mathbf{C})$ was done by KOSTERS [28], see also NAIMARK [34, ch. 3, §9]. Hopefully, an extension to $SO_0(n, 1)$ and $SU(n, 1)$ is feasible.

The reader of this paper is supposed to already have a modest knowledge about certain elements of semisimple Lie theory, like principal series and spherical functions. Suitable references will be given. Some of this preliminary material can also be found in the earlier version [27]. Modern accounts of the infinitesimal approach to $SL(2, \mathbf{R})$ can be found, for instance, in SCHMID [36, §2] or VAN DIJK [9]. TAKAHASHI [42] also presented a global approach to $SL(2, \mathbf{R})$, partly based on an earlier version of the present paper, partly (the global proof of Theorem 5.4) independently.

Finally, I would like to thank G. van Dijk and M. Flensted-Jensen for useful comments.

2. THE CANONICAL MATRIX ELEMENTS OF THE PRINCIPAL SERIES

2.1. PRELIMINARIES

Let G be a locally compact group satisfying the second axiom of countability (lcsc. group). A *Hilbert representation* of G is a strongly continuous but not necessarily unitary representation τ of G on some Hilbert space $\mathscr{H}(\tau)$ (which is always assumed to be separable). Let K be a compact subgroup of G. A Hilbert representation τ of G is called *K-unitary* if the restriction $\tau|_K$ of τ to K is a unitary representation of K. A Hilbert representation τ of G is called *K-finite* respectively *K-multiplicity free* if τ is K-unitary and each $\delta \in \hat{K}$ has finite multiplicity respectively multiplicity 1 or 0 in $\tau|_K$. If τ is K-multiplicity free then the *K-content* $\mathscr{M}(\tau)$ of τ is the set of all $\delta \in \hat{K}$ which have multiplicity 1 in $\tau|_K$.

Let K be a compact abelian subgroup of G and let τ be a K-multiplicity free representation of G. Choose an orthogonal basis $\{\phi_\delta \mid \delta \in \mathscr{M}(\tau)\}$ of $\mathscr{H}(\tau)$ such that

$$\tau(k)\phi_\delta = \delta(k)\phi_\delta, \qquad \delta \in \mathscr{M}(\tau), k \in K.$$

We call $\{\phi_\delta\}$ a *K-basis* for $\mathscr{H}(\tau)$ and the functions $\tau_{\gamma\delta}(\gamma, \delta \in \mathscr{M}(\tau))$, defined by

$$\tau_{\gamma,\delta}(g) := (\tau(g)\phi_\delta, \phi_\gamma), \quad g \in G, \tag{2.1}$$

the *canonical matrix elements* of τ (with respect to K).