

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 28 (1982)  
**Heft:** 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** THE REPRESENTATION THEORY OF  $SL(2, \mathbb{R})$ , A NON-INFINITESIMAL APPROACH  
**Autor:** Koornwinder, Tom H.  
**Kurzfassung:** Abstract  
**DOI:** <https://doi.org/10.5169/seals-52233>

#### Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

#### Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

#### Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 10.08.2025

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

# THE REPRESENTATION THEORY OF $SL(2, \mathbf{R})$ , A NON-INFINITESIMAL APPROACH

by Tom H. KOORNWINDER

## ABSTRACT

The representation theory of  $SL(2, \mathbf{R})$  is developed by the use of non-infinitesimal methods. This approach is based on an explicit knowledge of the matrix elements of the principal series with respect to the  $K$ -basis. The irreducible subquotient representations of the principal series are determined, and also their Naimark equivalences and unitarizability. All irreducible  $K$ -unitary,  $K$ -finite representations of  $SL(2, \mathbf{R})$  are classified, where an inversion formula for the generalized Abel transform provides an important tool.

## 1. INTRODUCTION

In 1947 two papers appeared on the representation theory of the two prototypes of noncompact semisimple Lie groups, namely by BARGMANN [2] on  $SL(2, \mathbf{R})$  and by GELFAND & NAIMARK [18] on  $SL(2, \mathbf{C})$ . The methods in the two papers are surprisingly different. Bargmann uses the infinitesimal (i.e. Lie algebraic) approach, while Gelfand & Naimark prefer non-infinitesimal (global) methods. In subsequent work to generalize these results for arbitrary noncompact semisimple Lie groups, the Bargmann approach has proved to be most successful, in particular by the work of Harish-Chandra. (However, it is interesting to note MAUTNER's [31] review of HARISH-CHANDRA's paper [22].)

Without denying the success of the infinitesimal approach, I want to add some motivation for a paper which favours the global approach:

- (a) *The didactic argument.* The global approach is a more natural and direct one and it does not require so much sophisticated functional analysis as the infinitesimal approach.