

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 27 (1981)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE RIEMANN-ROCH THEOREM FOR COMPACT RIEMANN SURFACES
Autor: Simha, R. R.
Kapitel: §1. Introduction
DOI: <https://doi.org/10.5169/seals-51747>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 08.04.2026

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

THE RIEMANN-ROCH THEOREM FOR COMPACT RIEMANN SURFACES

by R. R. SIMHA

§ 1. INTRODUCTION

The aim of this article is to present a sheaf-theoretic proof of the Riemann-Roch theorem (including Serre duality) for vector bundles on compact Riemann surfaces. The basic assumption will be the finite dimensionality of cohomology vector spaces; no further potential theory will be used. Thus the proof will work (with trivial modifications) in the algebraic case also (over an algebraically closed field of any characteristic). The possibly new contribution of the article is a simple direct proof of the fact that the degree of the canonical divisor is $2g - 2$, where $g = \dim H^1(X, \mathcal{O})$.

We now give an outline of the contents. The rather long Section 2 gives the necessary definitions and sheaf-theoretic results, and the consequences of the finite dimensionality theorem which are needed later. Section 3 gives the preliminary form of the Riemann-Roch theorem. The identity $\deg K = 2g - 2$ is proved in Section 4. Serre duality and the final form of the Riemann-Roch theorem are proved in Section 5.

Our exposition borrows freely from those of Serre [5] and Mumford [4]. We should also mention the proof of the Riemann-Roch theorem given in Grauert-Remmert [1] (Ch. VII).

I thank the referee for his careful reading of the manuscript, which has eliminated many errors.

§ 2. LINE BUNDLES AND VECTOR BUNDLES. SHEAF THEORETIC PRELIMINARIES

In all that follows, X will denote a compact Riemann surface, i.e. a connected compact complex manifold of complex dimension 1; $\mathcal{O} = \mathcal{O}_X$ will denote its structure sheaf, i.e. the sheaf of germs of holomorphic functions on X .