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It is shown that for 2-satisfiable  $F$  in CNF, there exists a satisfiable subset of the clauses  $C_1, \dots, C_n$  in  $F$  which has  $hn$  members. Moreover, there is a polynomial algorithm to find such a set. On the other hand, for any  $h' > h$ , there is some 2-satisfiable  $F$  which contains no satisfiable subset of at least  $h' | F |$  members ( $| F |$  being the number of clauses in  $F$ ).

Let  $Z(a)$  be the set of CNF's such that each  $F$  in CNF has an interpretation satisfying  $a | F |$  clauses. The construction problem of  $Z(a)$  is to compute for each  $F$  in  $Z(a)$  an interpretation which satisfies at least  $a | F |$  clauses. In this terminology it is well-known that  $P = NP$  iff the construction problem of  $Z(1)$  is in  $P$ . The result mentioned above shows that the construction problem for 2-satisfiable CNF's in  $Z(h)$  is in  $P$ . Let now  $h'$  be an algebraic number such that  $1 \geq h' > h$ . A somewhat mysterious result is then given: the construction problem for all 2-satisfiable CNF's in  $Z(h')$  is in  $P$ , iff  $P = NP$ . In other words, the set of 2-satisfiable CNF's which belong to  $Z(h')$  is NP-complete.

Specker and his coauthor remark that under Cook's hypothesis (i.e.,  $P \neq NP$ ), there is a "quantum jump" at  $h$ , because at this point, the complexity of computation passes over from  $P$  to  $NP$  which is no longer polynomial under Cook's hypothesis. They do not mention whether they consider their result to be positive or negative evidence for Cook's conjecture. Over the years I have asked several experts why they believe in the conjecture and have failed to be convinced by the reasons they give. I continue to feel that our state of ignorance today is such that nothing is known to make  $P \neq NP$  seem more plausible than  $P = NP$ .

According to Specker, the most important implication of 1979a is to draw attention to the golden ratio: we should not expect to fulfill more than 61.8% of our wishes.

#### SPECKER'S MATHEMATICAL PUBLICATIONS (1949-79)

1. 1949a. Die erste Cohomologiegruppe von Überlagerungen und Homotopieeigenschaften dreidimensionaler Mannigfaltigkeiten. *Commentarii Mathematici Helvetici*, vol. 23, pp. 303-333. Promotionsarbeit for Doctor of Mathematics at ETH, June, 1948.
2. 1949b. Nicht konstruktiv beweisbare Sätze der Analysis. *Journal of symbolic logic*, vol. 14, pp. 145-158.
3. 1949c. Sur un problème de Sikorski. *Colloquium Mathematicum*, vol. 2, pp. 9-12.
4. 1950a. Endenverbände von Räumen und Gruppen. *Math. Annalen*, vol. 122, pp. 167-174.

5. 1950b. Additive Gruppen von Folgen ganzer Zahlen. *Portugaliae Mathematica*, vol. 9, pp. 131-140.
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8. 1954b. Verallgemeinerte Kontinuumshypothese und Auswahlaxiom. *Archiv der Mathematik*, vol. 5, pp. 332-337.
9. 1957a. Zur Axiomatik der Mengenlehre (Fundierungs- und Auswahlaxiom). *Zeitschr. f. math. Logik und Grundlagen d. Math.*, vol. 3, pp. 173-210. This and 1954b make up the 1951 Habilitationsschrift at ETH.
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18. 1964. (With Haim GAIFMAN). Isomorphism types of trees. *Proc. Am. Math. Soc.*, vol. 15, pp. 1-7.
19. 1965a. (With Simon KOCHEN). Logical structures arising in quantum theory. *Symposium on the theory of models*, Amsterdam, pp. 177-189.
20. 1965b. (With Simon KOCHEN). The calculus of partial propositional functions. *Proc. 1964 Congress for Logic, Methodology and Philos. of Sci.*, pp. 45-57.
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22. 1967b. The fundamental theorem of algebra in recursive analysis. *Proc. Symposium Zürich-Ruschlikon*, pp. 321-329.
23. 1968. (With L. HODES). Length of formulas and elimination of quantifiers I. *Contributions to math. logic*, Amsterdam, ed. K. Schütte, pp. 175-188.
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28. 1976c. (With G. WICK). Länge und Formeln, *ibid.*, pp. 182-217.
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