Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 27 (1981)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: SPECKER'S MATHEMATICAL WORK FROM 1949 TO 1979
Autor: Wang, Hao
Kapitel: Specker's Mathematical Publications (1949-79)
DOI: https://doi.org/10.5169/seals-51741

It is shown that for 2-satisfiable F in CNF, there exists a satisfiable subset of the clauses $C_1, \ldots, C_n$ in F which has hn members. Moreover, there is a polynomial algorithm to find such a set. On the other hand, for any $h' > h$, there is some 2-satisfiable F which contains no satisfiable subset of at least $h' \mid F \mid$ members ($\mid F \mid$ being the number of clauses in F).

Let $Z(a)$ be the set of CNF's such that each F in CNF has an interpretation satisfying $a \mid F \mid$ clauses. The construction problem of $Z(a)$ is to compute for each F in $Z(a)$ an interpretation which satisfies at least $a \mid F \mid$ clauses. In this terminology it is well-known that $P = \text{NP}$ iff the construction problem of $Z(1)$ is in P. The result mentioned above shows that the construction problem for 2-satisfiable CNF's in $Z(h)$ is in P. Let now h' be an algebraic number such that $1 \geqslant h' > h$. A somewhat mysterious result is then given: the construction problem for all 2-satisfiable CNF's in $Z(h')$ is in P, iff $P = \text{NP}$. In other words, the set of 2-satisfiable CNF's which belong to $Z(h')$ is NP-complete.

Specker and his coauthor remark that under Cook's hypothesis (i.e., $P \neq \text{NP}$), there is a "quantum jump" at h, because at this point, the complexity of computation passes over from P to NP which is no longer polynomial under Cook's hypothesis. They do not mention whether they consider their result to be positive or negative evidence for Cook's conjecture. Over the years I have asked several experts why they believe in the conjecture and have failed to be convinced by the reasons they give. I continue to feel that our state of ignorance today is such that nothing is known to make $P \neq \text{NP}$ seem more plausible than $P = \text{NP}$.

According to Specker, the most important implication of 1979*a* is to draw attention to the golden ratio: we should not expect to fulfill more than 61.8% of our wishes.

Specker's Mathematical Publications (1949-79)

1. 1949*a*. Die erste Cohomologiegruppe von Überlagerungen und Homotopieeigenschaften dreidimonsionaler Mannigfaltigkeiten. *Commentarii Mathematici Helvetici*, vol. 23, pp. 303-333. Promotionsarbeit for Doctor of Mathematics at ETH, June, 1948.
2. 1949*b*. Nicht konstruktiv beweisbare Sätze der Analysis. *Journal of symbolic logic*, vol. 14, pp. 145-158.
3. 1949*c*. Sur un problème de Sikorski. *Colloquium Mathematicum*, vol. 2, pp. 9-12.
4. 1950*a*. Endenverbände von Räumen und Gruppen. *Math. Annalen*, vol. 122, pp. 167-174.

5. 1950*b*. Additive Gruppen von Folgen ganzer Zahlen. *Portugaliae Mathematica*, vol. 9, pp. 131-140.
6. 1953. The axiom of choice in Quine's new foundations for mathematical logic. *Proc. Nat. Acad. Sci. U.S.A.*, vol. 39, pp. 972-975.
7. 1954*a*. Die antinomie der Mengenlehre. *Dialectica*. vol. 8, pp. 234-244. Antrittsvorlesung at the ETH.
8. 1954*b*. Verallgemeinerte Kontinuumshypothese und Auswahlaxiom. *Archiv der Mathematik*, vol. 5, pp. 332-337.
9. 1957*a*. Zur Axiomatik der Mengenlehre (Fundierungs- und Auswahlaxiom). *Zeitschr. f. math. Logik und Grundlagen d. Math.*, vol. 3, pp. 173-210. This and 1954*b* make up the 1951 Habilitationsschrift at ETH.
10. 1957*b*. Teilmengen von Mengen mit Relationen. *Commentarii Mathematici Helvetici*, vol. 31, pp. 302-314.
11. 1957*c*. Eine Verschärfung des Unvollständigkeitssatzes der Zahlentheorie. *Bull. Acad. Polonais des Sciences, cl. III*, vol. 5, pp. 1041-1045.
12. 1957*d*. Der Satz vom Maximum in der rekursiven Analysis. *Constructivity in mathematics* (Proc. of 1957 colloquium at Amsterdam), pp. 254-265.
13. 1958. Dualität. *Dialectica*, vol. 12, pp. 451-465.
14. 1960. Die Logik nicht gleizeitig entscheidbarer Aussagen. *Dialectica*, vol. 14, pp. 239-246.
15. 1961*a*. (With R. MacDowell). Modelle der Arithmetik. *Infinitistic Methods*, Pergamon, London, pp. 257-263.
16. 1961*b*. (With P. Erdös). On a theorem in the theory of relations and a solution of a problem of Knaster. *Colloquium Mathematicum*, vol. 8, pp. 1921.
17. 1962. Typical ambiguity. *Logic, methodology and philos. of sci.: Proc. of the 1960 Congress*, pp. 116-124.
18. 1964. (With Haim Gaifman). Isomorphism types of trees. *Proc. Am. Math. Soc.*, vol. 15, pp. 1-7.
19. 1965*a*. (With Simon Kochen). Logical structures arising in quantum theory. *Symposium on the theory of models*, Amsterdam, pp. 177-189.
20. 1965*b*. (With Simon Kochen). The calculus of partial propositional functions. *Proc. 1964 Congress for Logic, Methodology and Philos. of Sci.*, pp. 45-57.
21. 1967*a*. (With Simon Kochen). The problem of hidden variables in quantum mechanics. *J. math. and mechanics*, vol. 17, pp. 59-88.
22. 1967*b*. The fundamental theorem of algebra in recursive analysis. *Proc. Symposium Zürich-Ruschlikon*, pp. 321-329.
23. 1968. (With L. Hodes). Length of formulas and elimination of quantifiers I. *Contributions to math. logic*, Amsterdam, ed. K. Schütte, pp. 175-188.
24. 1969. Ramsey's theorem does not hold in recursive set theory. *Logic Colloquium '69*, pp. 439-442. The result of this paper has been presented at the Universities of Bristol and Manchester in March 1966.
25. 1975. Logic of propositions not simultaneously decidable. *The logico-algebraic approach to quantum mechanics*, ed. C. A. Hooker, pp. 135-140. (This is an English translation of item 14.)
26. 1976*a*. (With Volker Strassen). Einleitung, *Komplexität von Entscheidungsproblemen*, pp. 1-10. They are also the editors of the volume.
27. 1976*b*. Ein polynomialer Algorithmus zur Bestimmung unabhängiger Repräsentantensystem, *ibid.*, pp. 72-85.
28. 1976*c*. (With G. Wick). Länge und Formeln, *ibid.*, pp. 182-217.
29. 1978*a*. Die Entwicklung der axiomatische Mengenlehre. *Jber. d. Dt. Math.-Verein.*, vol. 81, pp. 13-21.

30. 1978*b*. Algorithmische Kombinatorik mit Kleinrechnern. *Elemente der Mathematik*, vol. 33, pp. 25-35.
31. 1979*a*. (With K. LIEBERHERR). Complexity of partial satisfaction. *Proc. 20th Symposium on Foundations of Computer Science*, Puerto Rico, 1979, 8 pp.
32. 1979*b*. Paul Bernays. *Logic Colloquium '78*, Amsterdam, 1979, pp. 381-389.

(Reçu le 9 septembre 1980)

Hao Wang

The Rockefeller University
Mathematics Department
1230 York Avenue
New York, N.Y. 10021