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Artikel: SPECKER'S MATHEMATICAL WORK FROM 1949 TO 1979

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notes that about half of Bernays' papers may be classified as philosophical: this difference between them may be partly due to the different historical periods in which they live.

Enjoyment of interaction

Among Specker's publications, several papers seem to have been stimulated primarily by the enjoyment of personal interaction. Thus the paper 1949*c* dealt with a problem of Sikorski who was visiting Zürich then, while the paper 1964 continued the study to more elaborate cases (3 and 18 in the above list). The papers 1957*b* and 1961*b* (10 and 16 in the above list) seem to belong to the class of papers which are provoked by the infinite supply of problems from Erdös. Paper 11 answers a problem raised by Mostowski. A most obviously playful paper is 1978*b* (30) which gives, for the recognition problem, the generating problem, and the counting problem of the partition of finite sets, algorithms programmable on the "toy" computer HP-25.

Several of these papers contain clever constructions which stimulate extensions and generalizations. For example, the paper 10 gives the Specker graph which shows:

$$\omega^3 \rightarrow (2, \omega^3)^2 \text{ and } \omega^3 \not\rightarrow (3, \omega^3)^2 .$$

This leads to the function $f(n)$ such that $f(n) < \omega$,

$$\omega^n \rightarrow (f(n)-1, \omega^3)^2 \text{ and } \omega^n \not\rightarrow (f(n), \omega^3)^2 .$$

Eva Nosal many years later showed that $f(n) = 2^{n-2} + 1$ for $n \geqslant 3$, *J. London Math. Soc. (2), 8* (1974), 306-310.

Topology and recursive analysis

It is interesting to observe that Specker's early papers of 1949 and 1950 have continued to interest mathematicians over the years. For example, the paper 2 gives a bounded increasing recursive sequence of rational numbers that does not converge to a recursive real number. In a recent paper by M. I. Kanovič, such sequences are called Specker sequences, and the complexity of "limit candidates" for a Specker sequence is studied with the result that the larger the complexity of the candidate, the closer it is to the actual (more recursive) limit. The reference is: *Soviet Math. Dokl.*, vol. 15 (1974), No. 1, pp. 299-303.

The three papers on recursive analysis are remarkable in staying away from more controversial issues concerning constructivity. In particular, the first paper is probably the earliest use of recursive functions to elucidate constructive analysis.

Of the three papers on topology, I can out of ignorance have very little to say. The paper 1950*b* was done when Specker visited the Institute for Advanced Study. In it he is able to make interesting contributions to group theory as a topologist. It contains elegant ideas which received further development, for instance, almost two decades later in G. Nöbeling, "Verallgemeinerung eines Satzes von Herrn E. Specker", *Inventiones math.*, *6* (1968), 41-55. Let F be the Abelian group of sequences of integers with $\{a_n\} + \{b_n\} = \{a_n + b_n\}$. Specker shows that every countable subgroup of F is free and that F contains a nonfree subgroup of cardinality aleph-one (hence, F itself is not free). Furthermore, let F_b be the subgroup of F consisting of all bounded sequences of integers. Then it is shown that every subgroup of F_b with cardinality aleph-one is a free group. This last theorem is generalized by Nöbeling to show that for an arbitrary set X (rather than just the set of integers), the group of all bounded sequences from X is free and possesses a characteristic basis.

Specker's interest in the group F and its subgroups is suggested by the problem of determining the algebraic structure of the first cohomogy group of an infinite complex, a problem studied in his dissertation 1949*a*. The paper 1950*a* considers end lattices and introduces what is known as Specker compactization which is investigated extensively, for instance, in Herbert Abels, Specker-Kompaktifizierung von lokal kompakten topologischen gruppen, *Math. Z.*, *135* (1974), 325-361.

An example of beginning with concrete problems

In March 1966, Specker lectured in England on his result that Ramsey's theorem does not hold in recursive set theory. Afterwards, he was persuaded to present it at the Logic Colloquium of 1969 (and publish it as 24 in the above list). More exactly, Specker proves that there is a recursive (Σ_0) partition of the 2-elements sets of natural numbers which possesses no recursively enumerable (Σ_1) infinite sets of indiscernibles and that for every recursive partition there is always a Δ_3 set of indiscernibles. This suggests