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ON COUSIN-I COMPLEX SPACES

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SUMMARY

We study the properties of a complex space such that every closed analytic subspace has the 1-Cousin property. Under mild hypotheses we prove that X is a Stein space.

G. Berg in [2] proved very easily the following result: Let X be an open subset of a two-dimensional Stein space Y. If in X every 1-Cousin problem is solvable, then X is a domain of holomorphy in Y. It is not difficult to modify his proof and obtain that, under the stated assumption, X is always a Stein space.

In this note we want to consider this problem also for higher dimensional complex spaces. The proofs are always by induction on the dimension; they are similar to the proofs in the two dimensional case. This is also a partial generalization of my previous [1], theorem 2. We say that a complex space X is Cousin-I or has the 1-Cousin property if every 1-Cousin problem on X is solvable. Any Stein space is a Cousin-I space. We denote by $O(X)$ the algebra of holomorphic functions on a complex space X. For each $f \in O(X)$ we put

$$\{x \in X : f(x) = 0\} =: V(f),$$

an analytic subset of X. In this paper we consider for simplicity only complex spaces with bounded Zariski tangent dimension.

LEMMA 1. *Let X be a complex space with the 1-Cousin property and $f \in O(X)$ such that for each $x \in X$ the germ of f around x is a non-zero divisor in $\mathcal{O}_{X,x}$. We put*

$$Z := (V(f), \mathcal{O}_Z)$$

with

$$\mathcal{O}_Z := \mathcal{O}_X / f\, \mathcal{O}_{X|V(f)}.$$

Then the natural map $p: O(X) \to O(Z)$ is surjective.

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