

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 27 (1981)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: LOCALLY HOMOGENEOUS VARIATIONS OF HODGE STRUCTURE
Autor: Zucker, Steven
Kurzfassung: TABLE OF CONTENTS
DOI: <https://doi.org/10.5169/seals-51751>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 15.04.2026

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

TABLE OF CONTENTS

1. Preliminaries	246
2. Vector bundles on $\Gamma \backslash M$	250
3. The cohomology groups $H^n(\Gamma; \rho, V)$	254
4. The variation of Hodge structure associated to (ρ, V)	261
5. Hodge theory for $H^n(\Gamma; \rho, V)$, from the variation of Hodge structure	264
REFERENCES	276

§1. PRELIMINARIES

Let G be a connected real semi-simple Lie group with finite center, K a maximal compact subgroup of G , and let $\mathfrak{g} \supset \mathfrak{k}$ be the corresponding Lie algebras. For any sub-algebra $\mathfrak{a} \subset \mathfrak{g}$, we put

$$\mathfrak{a}_{\mathbb{C}} = \mathfrak{a} \otimes_{\mathbb{R}} \mathbb{C}.$$

If B denotes the Killing form of \mathfrak{g} , B is negative-definite on \mathfrak{k} , and we let \mathfrak{p} denote the orthogonal complement under B of \mathfrak{k} in \mathfrak{g} . Then $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$ is a so-called Cartan decomposition of \mathfrak{g} , and B is positive-definite on \mathfrak{p} .

Let $M = G/K$, the corresponding symmetric space. As $[\mathfrak{k}, \mathfrak{p}] \subset \mathfrak{p}$, B defines an $(\text{Ad } K)$ -invariant inner product on \mathfrak{p} ; and since we may identify \mathfrak{p} naturally as the tangent space to M at the identity coset $x_0 = K$, B determines a unique Riemannian metric on M which is invariant under the canonical left G -action.

Assume initially that M is an *irreducible* symmetric space. Then, if one wishes, G can be taken to be a non-compact almost simple group (i.e., \mathfrak{g} is a simple Lie algebra). In that case, the space M admits a homogeneous complex structure, and becomes an *Hermitian* symmetric space, precisely when \mathfrak{k} has a non-trivial center \mathfrak{z} . In this case, $\dim \mathfrak{z} = 1$, and $Z = \exp \mathfrak{z}$ is the identity component of the center of K . Let G^{ad} denote the adjoint group of G (i.e., the automorphism group of M) and let $K^{\text{ad}}, Z^{\text{ad}}$ be the corresponding subgroups of G^{ad} . A choice of $z_0 \in Z^{\text{ad}}$ of order 4 (for which $\text{Ad}(z_0^2)$ is a Cartan involution of \mathfrak{g}) determines an almost-complex structure on \mathfrak{p} :