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**THEOREM.** *Let  $M = M(n_1, \dots, n_k)$  be a generalized flag manifold for which the Conjecture C holds. Then*

$$G(M) = \{[M]\}.$$

*In particular the Grassmann manifolds  $G_p(\mathbb{C}^{p+q})$  for  $p \neq q$  and the flag manifolds  $U(n)/T^n$  are all generically rigid.*

§1. GENUS AND SELF MAPS

Let  $P$  denote a fixed set of primes. Two  $P$ -sequences

$$S_1, S_2: P \rightarrow E(X_0)$$

are called *equivalent*, if there exist maps  $h(0) \in E(X_0)$  and

$$h(p) \in \text{im}(E(X_p) \xrightarrow{\text{can}} E(X_0))$$

such that for all  $p \in P$  one has

$$h(0) S_1(p) = S_2(p) h(p).$$

*Definition 1.1.* We denote by  $P\text{-Seq}(E(X_0))$  the set of equivalence classes of  $P$ -sequences in  $E(X_0)$ .

If  $P$  is a finite set of primes and  $X$  a nilpotent space of finite type, then there is a canonical map

$$\theta: G(X) \rightarrow P\text{-Seq}(E(X_0)).$$

It is defined as follows. Let  $Y \in G(X)$  and  $P = \{p_1, \dots, p_n\}$ . Then the localization  $Y_P$  is a pull-back of maps  $X_{p_i} \xrightarrow{\lambda_i} X_0$ , i.e.  $Y_P \simeq \text{hoinvlim} \{X_{p_i} \xrightarrow{\lambda_i} X_0\}$ . The maps  $\lambda_i$  induce equivalences  $\bar{\lambda}_i \in E(X_0)$  and we put

$$\theta(Y) = \{[\bar{\lambda}_1, \bar{\lambda}_2, \dots, \bar{\lambda}_n]\}.$$

If  $Y_P$  may also be represented by  $\text{hoinvlim} \{X_{p_i} \xrightarrow{\mu_i} X_0\}$ , then there exist maps  $h(0) \in E(X_0)$  and  $\tilde{h}(p_i) \in E(X_{p_i})$ ,  $i \in \{1, \dots, n\}$  rendering the diagrams

$$\begin{array}{ccc}
 X_{p_i} & \xrightarrow{\tilde{h}(p_i)} & X_{p_i} \\
 \lambda_i \downarrow & & \downarrow \mu_i \\
 X_0 & \xrightarrow{h(0)} & X_0
 \end{array}$$

homotopy commutative and thus inducing  $\text{hoinvlim } \{\lambda_i\} \simeq \text{hoinvlim } \{\mu_i\}$ . Hence

$$\{[\bar{\lambda}_1, \dots, \bar{\lambda}_n]\} = \{[\bar{\mu}_1, \dots, \bar{\mu}_n]\} \in P\text{-Seq}(E(X_0))$$

and therefore  $\theta$  is well defined.

LEMMA 1.2. Let  $X$  be a nilpotent space of finite type and let  $P$  denote a finite set of primes. Then

$$\theta: G(X) \rightarrow P\text{-Seq}(E(X_0))$$

is surjective with fibers of the form

$$\theta^{-1}(\theta(Y)) = \{Z \in G(X) \mid Z_P \simeq Y_P\}.$$

*Proof.* Let  $P = \{p_1, \dots, p_n\}$  and

$$\{[\bar{f}_1, \dots, \bar{f}_n]\} \in P\text{-Seq}(E(X_0)).$$

Let  $e_i: X_{p_i} \rightarrow X_0$  denote the canonical maps and put

$$f_i = \bar{f}_i \circ e_i: X_{p_i} \rightarrow X_0.$$

Define  $W = \text{hoinvlim } \{f_i\}$ ;  $W$  comes equipped with a canonical map  $f: W \rightarrow X_0$ . Let  $Z$  be the homotopy pull back of  $W \xrightarrow{f} X_0 \xleftarrow{\text{can}} X_{\bar{P}}$ , where  $\bar{P}$  denotes the set of primes complementary to  $P$ . Then  $Z \in G(X)$  and

$$\theta(Z) = \{[\bar{f}_1, \dots, \bar{f}_n]\};$$

thus  $\theta$  is surjective. It is clear from the definition of  $\theta$  that for  $Y, Z \in G(X)$  one has  $\theta(Y) = \theta(Z)$  if and only if  $Y_P \simeq Z_P$ .

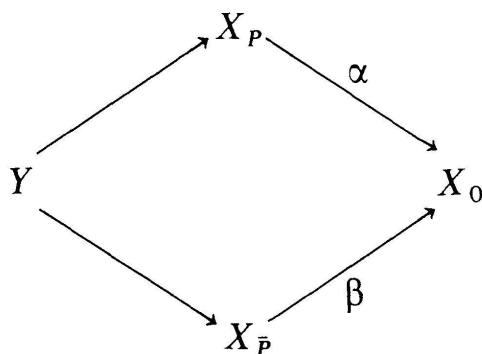
The next lemma provides a sufficient condition for  $\theta$  to be monic "at the basepoint".

LEMMA 1.3. Let  $X$  be a nilpotent space of finite type. Suppose that there exists a finite set of primes  $P$  with complement  $\bar{P}$  such that

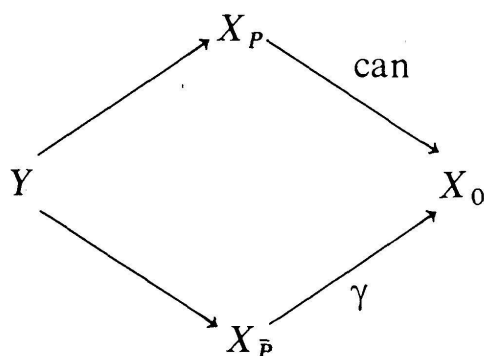
- a)  $Y \in G(X)$  implies  $Y_{\bar{P}} \simeq X_{\bar{P}}$
- b) every  $f \in E(X_0)$  can be written as  $f_1 \circ f_2$  with  $f_1 \in \text{im}(E(X_P) \xrightarrow{\text{can}} E(X_0))$  and  $f_2 \in \text{im}(E(X_{\bar{P}}) \rightarrow E(X_0))$ .

Then for  $\theta: G(X) \rightarrow P\text{-Seq}(E(X_0))$  as above, one has  $\theta^{-1}(\theta(X)) = \{X\}$ .

*Proof.* Let  $Y \in G(X)$  with  $\theta(Y) = \theta(X)$ . Then  $Y_P \simeq X_P$  by the definition of  $\theta$ , and  $Y_{\bar{P}} \simeq X_{\bar{P}}$  by assumption. Hence  $Y$  may be obtained as a homotopy pull back of the form



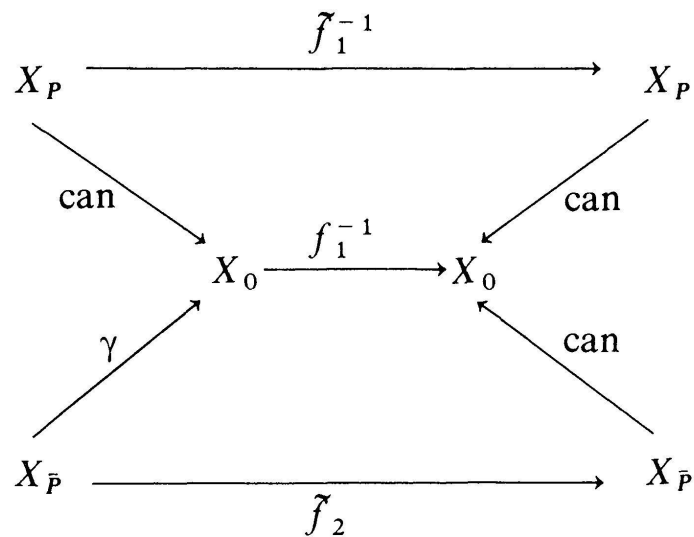
If  $\alpha$  induces  $\bar{\alpha} \in E(X_0)$  and if  $\gamma = \bar{\alpha}^{-1} \circ \beta$ , then  $Y$  is also a pull back of the form



Let  $\bar{\gamma} \in E(X_0)$  be the map induced by  $\gamma$  and write  $\bar{\gamma} = f_1 f_2$  with

$$f_1 \in \text{im}(E(X_P) \rightarrow E(X_0)), \quad f_2 \in \text{im}(E(X_{\bar{P}}) \rightarrow E(X_0)).$$

Choose a lift  $\tilde{f}_1^{-1} \in E(X_P)$  of  $f_1^{-1}$  and a lift  $\tilde{f}_2 \in E(X_{\bar{P}})$  of  $f_2$ . Then  $f_1^{-1} \bar{\gamma} = \text{can} \circ \tilde{f}_2$  and one can form a commutative diagram,



which shows that  $Y \simeq X$ .

## §2. THE CASE OF GENERALIZED FLAG MANIFOLDS

The following result is an easy consequence of [F].

LEMMA 2.1. Let  $M$  be a generalized flag manifold. Then the following holds.

- If  $g(\lambda) \in Gr(M_0)$  is a grading map with  $\lambda \in \mathbf{Z}_Q^*$  for some (not necessarily finite) set of primes  $Q$ , then  $g(\lambda)$  lifts to a homotopy equivalence  $\tilde{g}(\lambda): M_Q \rightarrow M_Q$ .
- Let  $P$  be an arbitrary set of primes with complement  $\bar{P}$ . Then every

$$f \in \langle Gr(M_0), N(H)/H \rangle$$

may be written in the form  $f = f_1 \circ f_2$  with

$$f_1 \in \text{im}(E(M_P) \rightarrow E(M_0))$$

and

$$f_2 \in \text{im}(E(M_{\bar{P}}) \rightarrow E(M_0)).$$

*Proof.* Let  $\lambda = k/l$  with  $k$  and  $l$  relatively prime integers. Then  $g(k)$  and  $g(l)$  lift to equivalences

$$\tilde{g}(k), \tilde{g}(l): M_Q \rightarrow M_Q.$$