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Autor: Jacobsen, Leif
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orthogonal action of G, the W' and W may already be chosen equivariant.

Similarly, it appears that there is a G-invariant version of the Stein extension theorem mentioned in remark 4 of [1]. This results from an invariant version of corollary 2 of [1], combined with the invariant Seeley extension theorem [7], p. 108. The X in the remark 4 is invariant if φ is (G acts on $\mathbf{R}^{n+1}$ by $g \,.\, (x, y) = (g \,.\, x, y)$ for $g \in G$).

An alternative approach would be via proposition 2 and the techniques of [4], which are somewhat similar.

We also wish to point out that there is a *G-invariant version of the Whitney spectral theorem* (see [6], ch. V or [9], ch. V):

Let $\Omega \subset \mathbf{R}^n$ be open and invariant under an orthogonal action of G, G compact Lie. Let $I \subset \mathscr{E}(\Omega)^G$ (using the action (3)) be an ideal. Then $f \in \mathscr{E}(\Omega)^G$ belongs to $\bar{I}$ if and only if for each $a \in \Omega$ there is a $g_a = g \in I$ such that $J_a(g) = J_a(f)$.

This goes via a fundamental lemma [6], p. 91, for the case $\Omega = a$ cube L. With the notations of that lemma, if L is replaced by $G \,.\, L$, K by $G \,.\, K$ and T_b^m by $Av_G \, T_b^m$, then $F \in \hat{I}$ may be assumed invariant on $G \,.\, L$, whence $| \tilde{\Phi} F - f |_{G.L}^m < \varepsilon$, ($\tilde{\Phi} = Av_G \, \Phi$) can be achieved. Then one proceeds. In the more general situation considered in [9], one needs [7], lemma 1.4.1 (p. 106).

The action (4) is adapted to the operators D^k. One might consider the simpler action on $J(X)$ ($X \subset \mathbf{R}^n$ G-invariant), given by $g \,.\, F = (F^k \circ g^{-1})_{k \in \mathbf{N}^n}$, for $F = (F^k)_{k \in \mathbf{N}^n}$, $g \in G$. The corresponding problem of finding f with $J(f) = F$, given $F \in \mathscr{E}(X)^G$, is now wholly different as simple examples show (e.g. $G = \mathbf{Z}_2$ acting by reflexion in $0 \in \mathbf{R}$). If f exists at all, it must have strong singularities on K. As may be gleaned from [3], there are topological restrictions on K, depending on G. It would perhaps be feasible to obtain some answers if new operators are used instead of the D^k.

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Leif Jacobsen

Matematiske Institut
Universitetsparken 5
2100 Kobenhavn
Danmark