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NORM AND SPECTRAL CHARACTERIZATIONS IN BANACH ALGEBRAS

by V. A. BELFI and R. S. DORAN

1. INTRODUCTION

Commutative Banach algebras enjoy a remarkably complete and beautiful structure theory due, in large part, to the efforts of a single man, I. M. Gelfand. The simplicity and beauty of this theory has prompted a number of mathematicians over the past fifteen years to seek conditions on an algebra (usually on its norm, spectral radius, etc.) in order that it be commutative.

Looking back to the late thirties, just before the splendid work of Gelfand, the question of characterizing those Banach algebras which are isomorphic to the real or complex numbers, or to the quaternions was being considered by S. Mazur. A corollary of one of his results, namely that a complex normed algebra which is a field is isomorphic to the complex numbers, turned out to be so basic that it is the foundation on which Gelfand's whole structure theory for commutative algebras rests.

Many papers have now been written concerning the characterization of commutativity in Banach algebras, and also on the problem of determining which algebras are isomorphic to $\mathbf{R}$, $\mathbf{C}$ or the quaternions $\mathbf{H}$. The purpose of this paper is to survey the results of these papers, giving full proofs (when possible), and also giving historical perspective. Our goal is to provide the reader, as best we are able, with an unobstructed view of the subject. To this end, and also to make the paper accessible to a wide audience, we have included a few well-known arguments from the general theory of Banach algebras. These serve, in several cases, to reveal important techniques which shed light on later developments. A substantial bibliography has been assembled to aid the reader wishing to pursue the subject further.