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Artikel: THE FAST SKEW-CLOSURE ALGORITHM
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For (i), define $B = A^{\Psi\Psi'\Psi}$

$$
\begin{aligned}
&= A^{\Phi\Phi'\Phi} && \text{by Theorem 5}\\
&\supseteq \bar{A}\bar{A}A \cup \bar{A}A \cup A && \text{by Lemma 1 (iii)}
\end{aligned}
$$

Therefore $A^{Q} \subseteq B^{\pi_1}$ from Lemma 2

For (ii) and (iii), let $B = A^{S}$

$$
\begin{aligned}
A^{R\Psi'\Psi} &= (I \cup A)^{\Phi'\Phi} && \text{by Theorem 5}\\
&\supseteq (I \cup \bar{A})(I \cup A) && \text{by Lemma 1 (ii)}\\
&\supseteq A \cup \bar{A}\\
&= B
\end{aligned}
$$

Also in this case, $A^{Q} \subseteq B^{\pi_1}$

In view of Theorem 5 and Lemma 1 (i) we have only to show now that $B^{\pi_1} \subseteq B^{\Phi\Phi'}$ to complete the proof. Using Lemma 3 repeatedly:

$$\pi_1 \subseteq \phi_1\pi_2 \subseteq \phi_1\phi_2\pi_3 \subseteq \ldots \subseteq \Phi\pi_{n+1}$$

But

$$X^{\pi_{n+1}} = \bar{X}X \cup X \subseteq X^{\Phi\Phi'}$$

therefore

$$\pi_1 \subseteq \Phi\Phi\Phi' = \Phi\Phi' \qquad \square$$

6. Conclusion

The close examination of a simple, practical matrix algorithm has led us to novel theoretical questions and to potentially useful generalizations of the algorithm. The principal contribution of this work to the programmer is the introduction of several very fast closure algorithms and the establishment of their correctness. The problems we have encountered in the theory of relations and closure operations have whetted our curiosity and suggest that further investigation may be rewarding.

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