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THE FAST SKEW-CLOSURE ALGORITHM ¹⁾

by M. J. FISCHER and M. S. PATERSON

ABSTRACT. A subtle matrix algorithm is explored and generalized. Originally used for transitive closures of symmetric Boolean matrices, this $O(n^2)$ algorithm computes a closure operation which is of interest for asymmetric and non-Boolean matrices too. The correctness of a generalized form of the algorithm is shown. The monoid generated by "skew-closure" and some of the more usual closures is investigated.

1. INTRODUCTION

The algorithm which forms the principal theme of this paper is of interest for several reasons. It is of mysterious ancestry; we have been unable to trace any published source which refers to it. It came to us by oral tradition at least seven years ago, when it impressed us with its speed and by the non-triviality of establishing its correctness. Further, whereas it seemed intended to be applied to Boolean matrices of symmetric and reflexive relations, the result of an application to more general matrices invited analysis.

The operation achieved by the algorithm we have termed "skew-closure". This closure is related to the more customary symmetric-and-transitive closure, and belongs to a very natural class of closure operations which we elaborate a little. In the interests of finding which matrix operations can be done equally rapidly, the monoid generated by several of these simple closures is treated in some detail. While this is finite, we later display a pair of slightly more complicated closures which together yield an infinite monoid.

¹⁾ Presented at the *Symposium über Logik und Algorithmik* in honour of Ernst SPECKER, Zürich, February 1980.