Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 26 (1980)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: LEVINE'S FORMULA IN KNOT THEORY AND QUADRATIC RECIPOCITY LAW

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Kapitel: §3. Proof of the main theorem

DOI: https://doi.org/10.5169/seals-51077

Otherwise

(15) $$\gamma_{\mathfrak{p}}^{\chi}(2q) = 1\,.$$

Remark. The condition on $\det \tilde{q}$ implies non-degeneracy of q at $\mathfrak{p}$.

§ 3. Proof of the main theorem

Note that the rank of q is even because determinant of the associated bilinear form is odd. Therefore

(16) $$\gamma_v^{\chi}(aq) = \left(\frac{a}{(\det \tilde{q})}\right) \gamma_v^{\chi}(q)$$

for *any* character χ.

Now let us apply the Weil reciprocity law for the character χ with support in dyadic components equal to the integers in the corresponding ring, and to the forms q and $2q$.

We have

$$\prod_v \gamma_v^{\chi}(2q) = 1$$

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For an archimedian components we have $\gamma_v^{\chi}(2q) = \gamma_v^{\chi}(q)$ because both depend only on the signatures. Therefore dividing those two identities, and using lemma 2 and (16) we obtain the identity (4).

Remark. Levine's lemma which in a specialization of the theorem for $R = \mathbf{Z}$ in fact follows from Milgram's formula (12). We should not worry about ramification. Therefore lemma 1 can be used for the character χ_0 and is actually a classical property of Gauss sums ([2]). Lemma 2 in this case essentially contains in [1].

§ 4. Proof of the lemmas

Proof of lemma 1. The Witt group of quadratic forms over a field of zero characteristic is generated by one-dimensional forms ([4]). Because γ^{χ} is a character of the Witt group it is enough to check the lemma for forms of one variable. Let π be a local parameter. Suppose that $q(x) = \alpha\, \pi^b\, x^2$,