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Otherwise

$$(15) \quad \gamma_p^\chi(2q) = 1.$$

Remark. The condition on $\det \tilde{q}$ implies non-degeneracy of q at p .

§ 3. PROOF OF THE MAIN THEOREM

Note that the rank of q is even because determinant of the associated bilinear form is odd. Therefore

$$(16) \quad \gamma_v^\chi(aq) = \left(\frac{a}{(\det \tilde{q})} \right) \gamma_v^\chi(q)$$

for any character χ .

Now let us apply the Weil reciprocity law for the character χ with support in dyadic components equal to the integers in the corresponding ring, and to the forms q and $2q$.

We have

$$\prod_v \gamma_v^\chi(2q) = 1$$

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For an archimedean components we have $\gamma_v^\chi(2q) = \gamma_v^\chi(q)$ because both depend only on the signatures. Therefore dividing those two identities, and using lemma 2 and (16) we obtain the identity (4).

Remark. Levine's lemma which in a specialization of the theorem for $R = \mathbf{Z}$ in fact follows from Milgram's formula (12). We should not worry about ramification. Therefore lemma 1 can be used for the character χ_0 and is actually a classical property of Gauss sums ([2]). Lemma 2 in this case essentially contains in [1].

§ 4. PROOF OF THE LEMMAS

Proof of lemma 1. The Witt group of quadratic forms over a field of zero characteristic is generated by one-dimensional forms ([4]). Because γ^χ is a character of the Witt group it is enough to check the lemma for forms of one variable. Let π be a local parameter. Suppose that $q(x) = \alpha \pi^b x^2$,