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In the first sum we use the bound $|G_T(x/i)| \leqslant 3T/2$, which is easily deduced from (4); for the second sum we calculate $G_T(y)$ directly for $T - 1 \leqslant y < TK$, and let $U = U(T, K)$ and $L = L(T, K)$ denote the upper and lower bounds for G_T on this interval.

Combining the estimates we obtain

$$(5)\quad \psi(x) - \psi\left(\frac{x}{T-1}\right) - \frac{3}{2}T\psi\left(\frac{x}{TK}\right) + L\left\{\psi\left(\frac{x}{T-1}\right) - \psi\left(\frac{x}{TK}\right)\right\} \leqslant xm_1(T) + O(T\log ex),$$

$$(6)\quad \psi(x) - \psi\left(\frac{x}{T-1}\right) + \frac{3}{2}T\psi\left(\frac{x}{TK}\right) + U\left\{\psi\left(\frac{x}{T-1}\right) - \psi\left(\frac{x}{TK}\right)\right\} \geqslant xm_1(T) + O(T\log ex).$$

We give an upper estimate of $\psi(x)/x$ using (5) with $T = 100$, $TK = 8911$, $L \geqslant -4.9054$, $m_1(100) \leqslant 1.00104$, and Chebyshev's bound $\limsup \psi(x)/x < 1.1056$. We find that $\limsup \psi(x)/x < 1.085$. We estimate $\psi(x)/x$ from below by using (6) with $T = 101$, $TK = 17749$, $U \leqslant 7.2930$, $m_1(101) \geqslant 1.00134$ and the preceding upper estimate of $\psi(x)/x$. We find that $\liminf \psi(x)/x > .924$.

Might Chebyshev have improved his bounds for $\psi(x)/x$ if he had used this method? We must report that that is quite unlikely, because considerable calculation was needed to obtain the modest improvement we have achieved.

NOTE ADDED IN PROOF. Diamond and Kevin Mc Curley have found another sharp elementary estimation method. Their article "Constructive elementary estimates for $M(x)$" will appear in *Number Theory — Proceedings of a conference held at Temple University, May 1980*, Lectures Notes in Math., Springer-Verlag, Berlin.

REFERENCES

[1] CHEBYSHEV, P. L. Mémoire sur les nombres premiers. *J. Math. Pure Appl. 17* (1852), 366-390. Also appears in *Mémoires présentés à l'Académie Impériale des Sciences de St.-Pétersbourg par divers Savants et lus dans ses Assemblées 7* (1854), 15-33. Also appears in *Œuvres* v. 1 (1899), 49-70.

[2] INGHAM, A. E. *The distribution of prime numbers*. Cambridge Tracts, No. 30, Cambridge, 1932. Reprnted by Hafner, New York, 1971.

[3] LANDAU, E. *Handbuch der Lehre von der Verteilung der Primzahlen*. Teubner, Leipzig, 1927. Reprinted with an appendix by P. T. Bateman, Chelsea, New York, 1953.

[4] MATHEWS, G. B. *Theory of Numbers*. Part 1, Deighton, Bell, Cambridge, 1892.

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