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14. VANISHING CYCLES

Let f be a germ in \mathcal{F} , and let \bar{f} be a nearby Morse function with μ distinct critical values t_1, \dots, t_μ in the disk D_δ^2 of radius δ about 0 in \mathbb{C} . A path α_i in $D_\delta^2 - \{t_1, \dots, t_\mu\}$ from δ to t_i determines (up to sign) a *vanishing cycle* δ_i in $H_n(F)$. The self-intersection (δ_i, δ_i) is $2(-1)^{n/2}$ or 0 according as n is even or odd. Choose paths $\alpha_1, \dots, \alpha_\mu$ in $D_\delta^2 - \{t_1, \dots, t_\mu\}$ from δ to t_1, \dots, t_μ respectively, such that the union of the images of the paths is a deformation retract of D_δ^2 ; then the corresponding vanishing cycles $\delta_1, \dots, \delta_\mu$ are a basis of $H_n(F)$ [Brieskorn 4, Appendix]. The basis $\delta_1, \dots, \delta_\mu$ is called an *ordered* (or *distinguished*) *basis of vanishing cycles* if t_1, \dots, t_μ are ordered so that the loop going once counter-clockwise around the boundary of D_δ^2 is homotopic in $\pi_1(D_\delta^2 - \{t_1, \dots, t_\mu\}, \delta)$ to the product $\beta_1 * \dots * \beta_\mu$, where β_i is the loop going out α_i almost to t_i , around t_i counter-clockwise, and back along α_i . References for this are [Gabrielov 1, Lamotke, Durfee 1].

Choose an ordered basis of vanishing cycles $\delta_1, \dots, \delta_\mu$ for the intersection pairing $(,)$ of $f(z_0, \dots, z_n) + z_{n+1}^2 + \dots + z_m^2$, where $m \equiv 2 \pmod{4}$. The *quadratic form diagram* of f with respect to the basis $\delta_1, \dots, \delta_\mu$ has vertices v_1, \dots, v_μ and edges from v_i to v_j if $(\delta_i, \delta_j) \neq 0$, weighted by (δ_i, δ_j) if $(\delta_i, \delta_j) \neq 1$. This diagram is connected [Lazzeri; Gabrielov 2]. It determines all the topological information in the singularity if $n \neq 2$ [Durfee 1]. There are a number of methods of computing these diagrams [A'Campo 2I; Gabrielov 3; Gusein-Zade]. The quadratic form diagrams of the germs of Table 2 are listed in column 5. Lemma 12.1 can be strengthened to show that if f topologically degenerates to g , then some quadratic form diagram for f is a subdiagram of some quadratic form diagram for g [Siersma, p. 82].

Characterization B7. There is an ordered basis of vanishing cycles for f such that the quadratic form diagram is a (weighted) tree.

It is shown in [A'Campo 2II] that Characterizations B1 and B7 are equivalent. In fact, the quadratic form diagrams for the germs in Table 2a are the same as the graph of their minimal resolutions (column 3 of Table 1).

15. THE MONODROMY GROUP

Let f be a germ in \mathcal{F} , and as above choose an ordered basis $\delta_1, \dots, \delta_\mu$ of vanishing cycles for $H_m(F)$, where F is the Milnor fiber of

$$f(z_0, \dots, z_n) + z_{n+1}^2 + \dots + z_m^2$$