Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 25 (1979)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: FIFTEEN CHARACTERIZATIONS OF RATIONAL DOUBLE POINTS AND SIMPLE CRITICAL POINTS

Autor: Durfee, Alan H.

Kapitel: 14. Vanishing cycles

DOI: https://doi.org/10.5169/seals-50375

14. Vanishing cycles

Let f be a germ in $\mathscr{F}$, and let $\bar{f}$ be a nearby Morse function with μ distinct critical values $t_1, ..., t_u$ in the disk D_δ^2 of radius δ about 0 in **C**. A path α_i in $D_\delta^2 - \{t_1, ..., t_u\}$ from δ to t_i determines (up to sign) a *vanishing cycle* δ_i in $H_n(F)$. The self-intersection (δ_i, δ_i) is $2(-1)^{n/2}$ or 0 according as n is even or odd. Choose paths $\alpha_1, ..., \alpha_\mu$ in $D_\delta^2 - \{t_1, ..., t_\mu\}$ from δ to $t_1, ..., t_\mu$ respectively, such that the union of the images of the paths is a deformation retract of D_δ^2; then the corresponding vanishing cycles $\delta_1, ..., \delta_\mu$ are a basis of $H_n(F)$ [Brieskorn 4, Appendix]. The basis $\delta_1, ..., \delta_\mu$ is called an *ordered* (or *distinguished*) *basis of vanishing cycles* if $t_1, ..., t_\mu$ are ordered so that the loop going once counter-clockwise around the boundary of D_δ^2 is homotopic in $\pi_1(D_\delta^2 - \{t_1, ..., t_\mu\}, \delta)$ to the product $\beta_1 * ... * \beta_\mu$, where β_i is the loop going out α_i almost to t_i, around t_i counter-clockwise, and back along α_i. References for this are [Gabrielov 1, Lamotke, Durfee 1].

Choose an ordered basis of vanishing cycles $\delta_1, ..., \delta_\mu$ for the intersection pairing (,) of $f(z_0, ..., z_n) + z_{n+1}^2 + ... + z_m^2$, where $m \equiv 2 \pmod 4$ The *quadratic form diagram* of f with respect to the basis $\delta_1, ..., \delta_\mu$ has vertices $v_1, ..., v_\mu$ and edges from v_i to v_j if $(\delta_i, \delta_j) \neq 0$, weighted by (δ_i, δ_j) if $(\delta_i, \delta_j) \neq 1$. This diagram is connected [Lazzeri; Gabrielov 2]. It determines all the topological information in the singularity if $n \neq 2$ [Durfee 1]. There are a number of methods of computing these diagrams [A'Campo 2I; Gabrielov 3; Gusein-Zade]. The quadratic form diagrams of the germs of Table 2 are listed in column 5. Lemma 12.1 can be strengthened to show that if f topologically degenerates to g, then some quadratic form diagram for f is a subdiagram of some quadratic form diagram for g [Siersma, p. 82].

Characterization B7. There is an ordered basis of vanishing cycles for f such that the quadratic form diagram is a (weighted) tree.

It is shown in [A'Campo 2II] that Characterizations B1 and B7 are equivalent. In fact, the quadratic form diagrams for the germs in Table 2a are the same as the graph of their minimal resolutions (column 3 of Table 1).

15. The monodromy group

Let f be a germ in $\mathscr{F}$, and as above choose an ordered basis $\delta_1, ..., \delta_\mu$ of vanishing cycles for $H_m(F)$, where F is the Milnor fiber of

$$f(z_0, ..., z_n) + z_{n+1}^2 + ... + z_m^2$$