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Characterization B5. The quadratic form of f is negative definite.

The equivalence of Characterizations B1 and B5 is proved in [Tjurina 1]. By explicit computation the quadratic forms of the germs in Table 2a are shown to be negative definite, and those of Table 2b are shown to be negative semi-definite. (In fact, the quadratic form of a germ in Table 2a is isomorphic to the intersection pairing of its minimal resolution, and the quadratic form of a germ of type $\tilde{E}_k$ in Table 2b is isomorphic to the quadratic form of E_k plus a two-dimensional zero form.) The result then follows from Proposition 10.1 and Lemma 12.1. When $n = 2$, the Milnor fiber F is in fact diffeomorphic to the minimal resolution M of $f^{-1}(0)$, since the singularity of $f^{-1}(0)$ is an absolutely isolated double point [Brieskorn 1, Theorem 4; Tjurina 1, Lemma 1].

When $n = 2$, the equivalence of Characterizations A2 and B5 follows from the following result [Durfee 2, Proposition 3.1].

THEOREM 12.2. *Twice the geometric genus p of $f^{-1}(0)$ equals the number of positive plus the number of zero diagonal elements in a diagonalization of the intersection pairing over the real numbers.*

The classification of germs according to signature of the quadratic form has been extended in [Arnold 3]; see also [Durfee 2, Proposition 3.3].

13. Nearby Morse functions

A *deformation* of a germ $f \in \mathcal{F}$ is a germ $g : \mathbf{C}^{n+1} \times \mathbf{C} \to \mathbf{C}$ with $g(z, 0) = f(z)$. Choose ε and δ for f as in §11. Then choose $\eta > 0$ such that for all $|t| < \eta$ and $|\delta'| \leqslant \delta$, the set $\{z \in \mathbf{C}^{n+1} : g(z, t) = \delta'\}$ intersects S_ε^{2n+1} transversally and the critical values of $g(z, t)$ for fixed t are less than δ in absolute value. A germ $\bar{f}$ is a *nearby Morse function* to f if $\bar{f}$ has only non-degenerate critical points in D_ε^{2n+2} and there is a deformation g and a t_0 with $|t_0| < \eta$ such that $\bar{f}(z) = g(z, t_0)$.

Characterization B6. There is a nearby Morse function to f with one or two critical values.

In fact, the nearby Morse function has one critical value if and only if f is right equivalent to A_2, since the quadratic form diagram is connected (§14). This surprising characterization is in [A'Campo 2II], where it is shown that Characterization B1 implies B6, and B6 implies B7 below.