Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 25 (1979)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: FIFTEEN CHARACTERIZATIONS OF RATIONAL DOUBLE POINTS AND SIMPLE CRITICAL POINTS
Autor: Durfee, Alan H.
Kapitel: 11. Simple germs and moduli
DOI: https://doi.org/10.5169/seals-50375

(iii) *The germs in Table 2c right simplify only to the germs in Table 2b and 2a.*

(iv) *A germ in $\mathscr{F}$ not right equivalent to a germ in Table 2a, b, or c right simplifies to a germ in Table 2c.*

11. Simple germs and moduli

A germ $f \in \mathfrak{m}$ is said to be *right* (or *contact*) *simple* if there is a neighborhood of f in $\mathfrak{m}$ intersecting only finitely many right (or contact) equivalence orbits. In the language of algebraic geometry, a germ f is contact simple if and only if the versal deformation of $f^{-1}(0)$ contains only finitely many isomorphism classes of analytic spaces.

The germs in Table 2a are right and contact simple by Proposition 10.1. The germs in Table 2b are not contact simple (and hence not right simple): $\tilde{E}_6$ is a family of cones over non-singular elliptic curves in $\mathbf{C}P^2$, $\tilde{E}_7$ is a family of four lines through the origin in $\mathbf{C}^2$, and $\tilde{E}_8$ is a family of three parabolas [Arnold 1; Siersma]. Note that the germs of Table 2c form one-dimensional families under right equivalence, but all members of the family are contact equivalent [Laufer 4; Siersma p. 54]. Clearly if a germ g right simplifies to f and f is not right simple, then g is not right simple; the same applies to contact equivalence.

Characterization B3. The germ f is right simple.

Characterization B4. The germ f is contact simple.

The equivalence of Characterizations B1 and B3 follows from Proposition 10.1 and the above remarks [Arnold 1]. Characterization B3 implies Characterization B4 by definition. Conversely, a contact simple germ f which is not right simple right simplifies to a germ in Table 2b (by Proposition 10.1), but these are not contact simple. Hence f must be right simple.

The classification of simple germs has recently been extended to complete intersections [Giusti]. The *modality* of a germ f is defined in [Arnold 3]. A right-simple germ is zero-modal; all right equivalence classes of 1 and 2-modal germs have been listed [Arnold 2, 3, 5]. Moduli of resolutions of two-dimensional normal singularities are studied in [Laufer 3, 4]. The following result provides a connection between Characterizations A2 and B3.

Theorem 11.1 [Randell]. *For almost all germs $f(x, y, z)$ (in the sense of the Newton diagram), the geometric genus p of $f^{-1}(0)$ is less than or equal to the modality of f.*