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**Autor:** Maltese, George

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*Application 4* (The spectral theorem for normal operators). Let  $\mathcal{H}$  be a Hilbert space and let  $\mathcal{L}(\mathcal{H})$  denote the Banach algebra of all bounded linear operators on  $\mathcal{H}$ . Consider a subalgebra  $A \subset \mathcal{L}(\mathcal{H})$  with the following properties:

- (i)  $A$  is commutative;
- (ii)  $A$  is closed;
- (iii) If  $T \in A$  then  $T^* \in A$ ;
- (iv) The identity operator belongs to  $A$ .

Let  $\Delta$  denote the maximal ideal space of  $A$ . Since each  $T \in A$  is normal it follows that  $\|T\| = \|\hat{T}\|_\infty$  for every  $T \in A$ .

For each pair of vectors  $\xi, \eta \in \mathcal{H}$  define a mapping  $L_{\xi, \eta} : A \rightarrow \mathbb{C}$  by

$$L_{\xi, \eta}(T) = (T\xi, \eta)$$

then we have

$$|L_{\xi, \eta}(T)| \leq \|T\| \cdot \|\xi\| \|\eta\| = \|\xi\| \|\eta\| \cdot \|\hat{T}\|_\infty$$

Therefore by Theorem 1 there exists a bounded complex Radon measure  $\mu_{\xi, \eta}$  on  $\Delta$  such that  $\|\mu_{\xi, \eta}\| \leq \|\xi\| \cdot \|\eta\|$  and

$$L_{\xi, \eta}(T) = \int_{\Delta} \hat{T} d\mu_{\xi, \eta} \quad \text{for every } T \in A.$$

An application of the Gelfand-Neumark theorem establishes the uniqueness of the measure. The usual construction of a unique resolution of the identity on the Borel sets of  $\Delta$  can be made based on this formula. A specialization of this formula to a single normal operator leads to the classical spectral theorem. We shall not give the details here since many excellent accounts exist (c.f. Berberian [1], [2], Segal-Kunze [18]). An especially lucid presentation is given in Rudin [16].

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George Maltese  
Universität Münster  
Münster, Germany