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INTEGRAL REPRESENTATION THEOREMS VIA BANACH ALGEBRAS

by George Maltese

1. Introduction

Many classical integral representation theorems of analysis can be obtained as special cases of the Choquet Representation Theorem [6], [7], [14] or the Krein-Milman Theorem. The procedure involves the definition of a suitable convex compact set in some locally convex space and an explicit description of the extreme points of this set. The latter is often a non-trivial task, therefore it seems appropriate to develop alternative methods which are general enough to yield a class of integral representation theorems. In many situations in which an integral representation formula is sought, there is a natural commutative Banach algebra inherent in the background. For example in the case of Bochner's theorem for positive definite functions on a locally compact abelian group G, the natural Banach algebra is the convolution algebra $L^1(G)$. In the case of the Schoenberg-Eberlein theorem for Fourier-Stieltjes transforms on locally compact abelian groups, the Banach algebra is again the convolution algebra. In the case of the Spectral Theorem for a normal operator T on a Hilbert space $\mathscr{H}$, the natural Banach algebra is the closed commutative * algebra generated by T and the identity operator.

In this paper we show that the above mentioned theorems are all special cases of a general result (Theorem 1) on the integral representation of certain linear forms defined on commutative Banach algebras. Specialization of Theorem 1 to symmetric Banach algebras yields a generalized version (Theorem 2) of a result of Raikov [10] for positive functionals on such algebras.

The proof of Theorem 1 is straight forward and its version for positive functionals on involution algebras is classical [11]. The main point here is the relative ease of application of Theorem 1 to a variety of situations.