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UNIVALENT FUNCTIONS, SCHWARZIAN DERIVATIVES AND QUASICONFORMAL MAPPINGS[1]

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1. INTRODUCTION

Univalent functions have been a popular topic in complex analysis for over sixty years. It has also been known for a long time that there are interesting connections between univalence and the Schwarzian derivative. More recently, one has discovered in this interplay the important role of quasiconformal mappings which not only provide a tool but, somewhat surprisingly, are intrinsic in the problem of deducing univalence from the behavior of the Schwarzian. In this survey, we shall describe some recent developments in this area.

After defining plane quasiconformal mappings, we briefly discuss quasicircles in Section 3. These curves, introduced by Pfluger [15] in 1960, play a central role in this survey. Section 4 deals with the problem of measuring the deviation of a simply connected domain A from a disc D by means of the Schwarzian derivative of the conformal mapping function $f: A \to D$. The starting point in Section 5 is the remarkable result that in a simply connected domain, a small Schwarzian derivative implies univalence if and only if the boundary of the domain is a quasicircle. The sufficiency of this condition is due to Ahlfors [1], the necessity to Gehring [2]. This result gives rise to considering the universal Teichmüller space, and in this way various explicit estimations for certain domain constants can be derived ([9]).

2. QUASICONFORMAL MAPPINGS

2.1 *Module of a curve family.* Roughly speaking, quasiconformal mappings are homeomorphisms under which conformal invariants remain quasi-invariant. A precise definition can be given, for instance, in terms of the module of curve families. Let A be a domain in the plane and Γ a family

[1]) Communicated to an International Symposium on Analysis, held in honour of Professor Albert Pfluger, ETH Zürich, 1978.