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Note also that $\mathcal{O}$ is holonomic as a $\mathcal{D}$-Module (one has $\mathcal{O} = \mathcal{D}1 \simeq \mathcal{D}/\Sigma\mathcal{D}\dfrac{\partial}{\partial x_i}$ and therefore char $(\mathcal{O}) = X$, the null section). This explains the interest of the following theorem, due again to Kashiwara [9], [11].

THEOREM 3.1. *If M and N are holonomic, then the sheaves* $\mathrm{Ext}^i_{\mathcal{D}}(M, N)$ *are* **C**-*analytically constructible* (i.e. *there exists a* **C**-*analytic stratification of X, such that, on each stratum, the sheaf is locally isomorphic to the constant sheaf* $\mathbf{C}^l$ *for some* l); *in particular, the fibers* $\mathrm{Ext}^i_{\mathcal{D}}(M, N)_x$ *are finite over* **C**.

Another problem is posed by the systematisation and extension to $\mathcal{D}$-Modules of the known theorems on regular connexions [5]. Here, one needs some regularity assumptions (for instance, the algebraic cohomology of a connection on an affine non-singular algebraic variety is the same as the analytic one when the connection is regular at infinity, but not in general). This subject is rapidly developing at the moment, and we will only mention some references:

a) In Kashiwara-Oshima [12], one will find regular $\mathcal{D}$- or $\mathcal{E}$-Modules, defined, and studied at generic points of the characteristic variety.

b) In Mebkhout [14] and Ramis [15], one will find systematic developments of the Grothendieck comparison theorem, in relation with $\mathcal{D}$-Modules and also with the "Cousin complex" of Grothendieck, in the analytical version of Ramis-Ruget [16].

Finally, I mention that, recently, Kashiwara and Kawai have announced an extension of the comparison theorem to any regular holonomic $\mathcal{D}$-Module.

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