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For that case, results similar to the one mentionned in this report have been obtained by Fuks-Segal (unpublished) and by T. Tsujishita [21].

For instance, when A is the algebra of smooth functions on M on which L_M acts by Lie derivative, their result is as follows. As it is described in § 3, the bundle E over M has a fiber F_n which is itself a principal U_n -bundle. Let us fix a fiber $F_n^\circ \approx U_n$ of this bundle; as it is invariant by the structural group $O_n \subset U_n$ of E , we get a subbundle E_o of E with typical fiber F_n° . Then $C^*(L_M, A)$ will be a model for the inverse image of E_o by the evaluation map $M \times \Gamma \rightarrow E$.

2. One of the most interesting problems is to know when, for a given class α in $H^*(L_M)$, there is a space X and a foliation F on $X \times M$ transverse to the fibers such that the image of α in $H^*(X)$ by the characteristic homomorphism (cf. 2) is non zero.

Very recent and remarkable results of Fuchs [23] show that this is the case for all classes coming from WSO_n . (For earlier partial results, see [4].) One might expect that his method will apply in general and show that the answer is affirmative for all classes in $H^*(L_M)$ (and also for the similar problem with $H^*(L_M; G)$).

There is also the problem of the possible continuous variations of characteristic classes for flat bundles which would be interesting to study (cf. [23]).

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