Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 24 (1978)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ON THE GELFAND-FUKS COHOMOLOGY
Autor: Haefliger, André
Kapitel: 8. Case of a manifold with boundary
DOI: https://doi.org/10.5169/seals-49696

Nutzungsbedingungen
Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Mehr erfahren

Conditions d'utilisation
L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. En savoir plus

Terms of use
The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. Find out more

Download PDF: 27.04.2026

ETH-Bibliothek Zürich, E-Periodica, https://www.e-periodica.ch

8. Case of a manifold with boundary

More generally we consider a closed manifold N of dimension p in a manifold M of dimension n. $L_{M,N}$ will denote the subalgebra of L_M of those vector fields on M which are tangent to N. An interesting particular case is when N is the boundary ∂M of M. For M compact, $L_{M,\partial M}$ can be considered as the Lie algebra of the group of diffeomorphisms of M.

First we consider the formal vector fields. *Let* $\mathfrak{a}_{n,p}$ *be the Lie subalgebra of formal vector fields on* R^n *which are tangent to* R^p identified to a linear subspace of R^n. Again $C^*(\mathfrak{a}_{n,p})$ denotes the DG-algebra of those multilinear alternate forms on $\mathfrak{a}_{n,p}$ depending only on finite order jets.

We describe a finite dimensional model for $C^*(\mathfrak{a}_{n,p})$. Let $E(h'_1, \ldots, h'_p, h''_1, \ldots, h''_{n-p})$ be the exterior algebra in generators h'_i and h''_j of degree $2i-1$. Let $R[c'_1, \ldots, c'_p, c_1'', \ldots, c''_{n-p}]^{\wedge}_{2p}$ be the quotient of the polynomial algebra in generators c'_i and c_i'' of degree $2i$ by the ideal of elements of degree $> 2p$.

Define

$$WU_{n,p} = E(h'_1, \ldots, h'_p, h''_1, \ldots, h''_{n-p}) \otimes R[c'_1, \ldots, c'_p, c''_1, \ldots, c''_{n-p}]^{\wedge}_{2p}$$

as the DG-algebra with differential defined by

$$dh_i' = c_i', \quad dh_i'' = c_i'', \quad dc_i' = 0, \quad dc_i'' = 0.$$

This is a model for the space $F_{n,p}$ obtained by restricting the universal principal $(U_p \times U_{n-p})$-bundle over the $2p$-skeleton of its basis represented by a product of Grassmanians with the usual even dimensional cell decomposition.

If $n \leqslant 2p$, $WU_{n,p}$ is also a model for a wedge of spheres. When $n > 2p$, it is a model for the product of the wedge of spheres corresponding to $WU_{2p,p}$ by $S^{2p+1} \times S^{2p+3} \ldots \times S^{2n-2p-1}$.

Theorem 1 (Koszul [11]). *There is a natural morphism*

$$WU_{n,p} \to C^*(\mathfrak{a}_{n,p})$$

inducing an isomorphism in cohomology.

As a consequence, $H^i(\mathfrak{a}_{n,p}) = 0$ for $0 < i \leqslant 2p$ and $i > p^2 + (n-p)^2 + 2p$. When $n \leqslant 2p$, the multiplication is trivial.

To have a model for the homomorphism induced by the inclusion of $\mathfrak{a}_{n,p}$ in $\mathfrak{a}_n$, we have the commutative diagramm

$$\begin{array}{ccc} C^*(\mathfrak{a}_n) & \longrightarrow & C^*(\mathfrak{a}_{n,p}) \\ \uparrow & & \uparrow \\ WU_n & \longleftarrow & WU_{n,p} \end{array}$$

where the second horizontal map sends h_i on $h_i' + h_i''$ and c_i on $c_i' + c_i''$ (by convention, h_i' or h_i'' is zero for $i > p$ or $i > n\text{-}p$, idem for c_i' and c_i''). Note that the natural map of theorem 1 should map the c_i' s and c_i'' not on the usual Chern classes defined by the connection but on the polynomials in Chern classes corresponding to $\sum x_k^i$, the Chern classes being the elementary symmetric functions in the formal variables x_k. These horizontal maps are also models for an inclusion of $F_{n,p}$ in F_n.

We consider again the bundle E over M associated to the tangent bundle of M and with fiber F_n. Its restriction above N contains a subbundle E' with fiber $F_{n,p}$.

THEOREM. *$C^*(L_{M,N})$ is a model for the space $\Gamma_{M,N}$ of continuous sections of the bundle E whose restriction to N have values in the subbundle E'.*

To make explicit computations, we construct a model for $\Gamma_{M,N}$, which will be finite dimensional in each degree when M and N have finite dimensional models. This is the purpose of the next paragraph.

9. CONSTRUCTION OF A MODEL FOR $C^*(L_{M,N})$

Consider the commutative diagramm of Lie algebras

$$\begin{array}{ccc} L_{M,N} & \longrightarrow & L_M \\ \downarrow & & \downarrow \\ L'_{M,N} & \longrightarrow & L'_M \end{array}$$

where L'_M and $L'_{M,N}$ are the quotients of L_M and $L_{M,N}$ by the subalgebra L_M^0 of vector fields on M whose infinite jet vanish at points of N.