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If G is the group SO_2 of rotations of S^1 , then $H(L_{S^1}; SO_2)$ is a model for $C^*(L_{S^1}; SO_2)$. It is generated by u and by an element e of degree 2 represented by

$$e(f, g) = \int_0^1 \begin{vmatrix} f & g \\ f' & g' \end{vmatrix} dx$$

The only relation is $eu = 0$.

2. CONNECTION WITH FOLIATIONS

Let me indicate very briefly the relation with characteristic classes of flat bundles (cf. [12]).

$H^*(L_M, G)$ could also be interpreted as the differentiable cohomology of a suitable differentiable category (for more informations see [4] and [15]).

We consider on the product $X \times M$ of a smooth manifold X with M a smooth foliation F whose leaves have the same dimension as X and cut each fibers $\{x\} \times M$ transversally.

To such a foliation is naturally associated a continuous DG -algebra map

$$\chi_F : C^*(L_M) \rightarrow \Omega_X$$

where Ω_X is the DG -algebra of differential forms on X . In fact there is a bijection between such morphisms and foliations F as above.

Passing to cohomology, we get the characteristic map

$$H^*(L_M) \rightarrow H^*(X; R)$$

If we replace the trivial bundle by a bundle E with fiber M , base space X and structural group G , then for a foliation F on E complementary to the fibers, we still get a morphism

$$\chi_F : C^*(L_M; G) \rightarrow \Omega_X$$

hence a characteristic homomorphism

$$H^*(L_M, G) \rightarrow H^*(X; R)$$

Denoting by BG the classifying space for G -bundles, we also have the usual characteristic map $H^*(BG; R) \rightarrow H^*(X; R)$. This map factorizes

through a map $H^*(BG; R) \rightarrow H^*(L_M; G)$ so that we get a commutative diagram

$$\begin{array}{ccc}
 & & H^*(L_M; G) \\
 & \nearrow & \downarrow \\
 H^*(BG; R) & & \\
 & \searrow & \\
 & & H^*(X; R)
 \end{array}$$

So it is important to compute the map $H^*(BG; R) \rightarrow H^*(L_M; G)$. When G is a compact connected Lie group, then $H^*(BG; R)$ is the algebra $I(G)$ of invariant polynomials on the Lie algebra of G , and the map from $I(G)$ to $C^*(L_M; G)$ is given by a G -connexion in $C^*(L_M)$ (cf. [5]).

In the example above, namely $M = S^1$ and $G = SO_2$, then $H^*(BSO_2)$ is a polynomial algebra in a generator of degree 2, the Euler class, which is mapped on a non zero multiple of e .

3. THE FORMAL VECTOR FIELDS AND THE DIAGONAL COMPLEX

Given a point x on M , we can consider the Lie algebra L_M^x of infinite jets at x of vector fields on M with the quotient topology. It is isomorphic to the Lie algebra \mathfrak{a}_n of formal vector fields $\sum v_i(x) \partial/\partial x^i$ in R^n , where the $v_i(x)$ are formal power series in the coordinates x^1, \dots, x^n .

The natural map $L_M \rightarrow L_M^x$ associating to a vector field its jet at x gives a DG -algebra morphism

$$C^*(L_M^x) \rightarrow C^*(L_M)$$

where $C^*(L_M^x)$ is the algebra of multilinear alternate forms on L_M^x depending only on finite order jets.

The first and most important step in the work of Gelfand-Fuks was the complete determination of the cohomology $H^*(\mathfrak{a}_n)$ of the topological Lie algebra of formal vector fields on R^n .

THEOREM 1. (Gelfand-Fuks [8], [9]). *Let $E(h_1, \dots, h_n)$ be the exterior algebra on generators h_i of degree $2i-1$ and let $R[c_1, \dots, c_n]_{2n}^\wedge$ be the quotient of the polynomial algebra in generators c_i of degree $2i$ by the ideal of elements of degree $> 2n$.*