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Artikel: ON A FUNCTIONAL EQUATION RELATING TO THE BRAUER-RADEMACHER IDENTITY

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and

$$R(p^{\beta}, p^{\alpha}) = \mu_A(p^{st}) \sum_{d \in A(pj^t)} f(d)\, h\left(\frac{n}{d}\right) = 0\,, \text{ since } s \geqslant 2\,.$$

Now, it is easy to see that $L(p^{\beta}, p^{\alpha}) = R(p^{\beta}, p^{\alpha})$ if and only if (16) holds. Thus the Theorem is proved.

COROLLARY *If* $f \in \mathscr{C}$ *and* $h = \mu_A g$, *where* $g \in \mathscr{M}$, *then the pair* (f, h) *satisfies the functional equation (14).*

Proof. We have

$$F_A(p^{\alpha}) f(p^{\alpha - t}) = \left\{ \sum_{d \in A(p^{st})} f(d)\, \mu_A(p^{st}/d)\, g(p^{st}/d) \right\} f(p^{(s-1)t})$$

$$= \left\{ \sum_{i=0}^{s} f(p^{it})\, \mu_A(p^{(s-i)t})\, g(p^{(s-i)t}) \right\} f(p^{(s-1)t})$$

$$= \{ f(p^{st}) - f(p^{(s-1)t})\, g(p^t) \} f(p^{(s-1)t})$$

$$(17) \qquad = f(p^{(2s-1)t}) - f(p^{(2s-2)t})\, g(p^t)\,,$$

since $f \in \mathscr{C}$. Also,

$$f(p^{\alpha}) F_A(p^{\alpha - t}) = f(p^{st}) \left\{ \sum_{i=0}^{s-1} f(p^{it})\, \mu_A(p^{(s-1-i)t})\, g(p^{(s-1-i)t}) \right\}$$

$$= f(p^{st}) \{ f(p^{(s-1)t}) - f(p^{(s-2)t})\, g(p^t) \}$$

$$(18) \qquad = f(p^{(2s-1)t}) - f(p^{(2s-2)t})\, g(p^t)\,.$$

Now, from (17) and (18), we see that (16) holds for all primes p and all integers $\alpha \geqslant 2$, where $t = \tau_A(p^{\alpha})$. Hence the Corollary follows in virtue of the Theorem.

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