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and

$$R(p^\beta, p^\alpha) = \mu_A(p^{st}) \sum_{d \in A(pjt)} f(d)\, h\left(\frac{n}{d}\right) = 0\,, \text{ since } s \geqslant 2\,.$$

Now, it is easy to see that $L(p^\beta, p^\alpha) = R(p^\beta, p^\alpha)$ if and only if (16) holds. Thus the Theorem is proved.

COROLLARY *If* $f \in \mathscr{C}$ *and* $h = \mu_A g$, *where* $g \in \mathscr{M}$, *then the pair* (f, h) *satisfies the functional equation (14).*

Proof. We have

$$\begin{aligned} F_A(p^\alpha) f(p^{\alpha-t}) &= \left\{ \sum_{d \in A(pst)} f(d)\, \mu_A(p^{st}/d)\, g(p^{st}/d) \right\} f(p^{(s-1)t}) \\ &= \left\{ \sum_{i=0}^{s} f(p^{it})\, \mu_A(p^{(s-i)t})\, g(p^{(s-i)t}) \right\} f(p^{(s-1)t}) \\ &= \{ f(p^{st}) - f(p^{(s-1)t})\, g(p^t) \} f(p^{(s-1)t}) \\ &= f(p^{(2s-1)t}) - f(p^{(2s-2)t})\, g(p^t)\,, \end{aligned} \tag{17}$$

since $f \in \mathscr{C}$. Also,

$$\begin{aligned} f(p^\alpha) F_A(p^{\alpha-t}) &= f(p^{st}) \left\{ \sum_{i=0}^{s-1} f(p^{it})\, \mu_A(p^{(s-1-i)t})\, g(p^{(s-1-i)t}) \right\} \\ &= f(p^{st}) \{ f(p^{(s-1)t}) - f(p^{(s-2)t})\, g(p^t) \} \\ &= f(p^{(2s-1)t}) - f(p^{(2s-2)t})\, g(p^t)\,. \end{aligned} \tag{18}$$

Now, from (17) and (18), we see that (16) holds for all primes p and all integers $\alpha \geqslant 2$, where $t = \tau_A(p^\alpha)$. Hence the Corollary follows in virtue of the Theorem.

REFERENCES

[1] BRAUER, A. und H. RADEMACHER. Aufgabe 31. *Jahresbericht der deutschen Mathematiker Vereinigung*, *35* (1926), pp. 94-95 (Supplement).

[2] COHEN, E. Some totient functions. *Duke Math. J.*, *23* (1956), pp. 515-522.

[3] —— Generalizations of the Eulerφ -function. *Scripta Math.*, *23* (1957), pp. 157-161.

[4] —— Representations of even functions (mod r), I. *Arithmetical identities*, *Duke Math. J.*, *25* (1958), pp. 401-421.

[5] —— Representations of even functions (mod r), II. Cauchy products. *Duke Math. J.*, *26* (1959), pp. 165-182.

[6] —— Arithmetical Inversion formulas. *Canadian J. Math.*, *12* (1960), pp. 399-409.

[7] —— The Brauer-Rademacher identity. *Amer. Math. Monthly*, *67* (1960), pp. 30-33.

[8] —— A trigonometric sum. *Math. Student*, *28* (1960), pp. 29-32.

[9] —— Arithmetical functions associated with the unitary divisors of an integer. *Math. Zeit.*, *74* (1960), pp. 66-80.

[10] DICKSON, L. E. *History of the theory of numbers, Vol. I.* Chelsea Publishing Company, New York, reprinted, 1952.

[11] MCCARTHY, P. J. Some remarks on arithmetical identities. *Amer. Math. Monthly, 67* (1960), pp. 539-548.

[12] —— Some more remarks on arithmetical identities. *Portugaliae Math., 21* (1962), pp. 45-57.

[13] —— Regular arithmetical convolutions. *Portugaliae Math., 27* (1968), pp. 1-13.

[14] NAGESWARA RAO, K. *Studies in Arithmetical functions.* Ph. D. thesis, 1967, University of Delhi, India.

[15] NARKIEWICZ, W. On a class of arithmetical convolutions. *Colloq. Math., 10* (1963), pp. 81-94.

[16] SHADER, E. L. The Unitary Brauer-Rademacher identity. *Atti. Acad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Natur., (8) 48* (1970), pp. 403-404; M.R. 42, #7583.

[17] SUBBARAO, M. V. The Brauer-Rademacher identity. *Amer. Math. Monthly, 72,* (1965), pp. 135-138.

[18] SZÜSZ, P. Once more the Brauer-Rademacher identity. *Amer. Math. Monthly, 74* (1967), pp. 570-571.

[19] VASU, A. C. A generalization of Brauer-Rademacher identity. *Math. Student, 33* (1965), pp. 97-101.

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