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7. The Background in Homotopy

Group theory, in fact had been present in combinatorial topology from the beginning, in the study of the fundamental group (the Poincaré group) of a space or in particular of a manifold. The fundamental group Π_1 for a polyhedron P naturally comes with a presentation of the form $\Pi_1 = F/R$, where F is a suitable free group generated by circuits in the one-skeleton of P, while its subgroup R is described from the 2-cells of P. Hence *this* sort of presentation was ready at hand for Hopf's study of the influence of the fundamental group—and his paper does make reference to the work of Reidemeister, one of the German topologists concerned with the fundamental group.

The introduction of the higher homotopy group was more recent. At the 1932 International Congress of Mathematicians in Zurich, E. Čech had described our present two-dimensional homotopy group in a very brief note. He wrote no further on the subject. Folklore has it that other topologists at the conference discouraged him from further work, pointing out that his Π_2 was an abelian group, while all the experience with Π_1 indicated that what was wanted was a non-abelian group. Hence the real credit for the higher homotopy groups goes to W. Hurewicz, who introduced them in several brief notes in 1935-36, together with proofs of several of their properties—enough to show that these higher homotopy groups *did* have utility in topology. In particular, his 1936 theorem *that* the homology groups of an aspherical polyhedron are determined by the fundamental group of that polyhedron is the exact starting point of our subject.

Other developments at this time emphasized the importance of homotopy —Hopf's discovery [1931] of the essential maps of S^3 on S^2, and the work of Whitehead on combinatorial homotopy. It was clearly the right time to investigate the relation between homotopy and homology.

8. The Cohomology of Groups

Once launched by topology, the higher dimensional cohomology groups of a group took on a life of their own. Eilenberg-Mac Lane and Mac Lane separately examined properties of the group $H^n(G, A)$ for a general G-module A. They found (from the study of Baer) the purely group-theoretical interpretation of $H^3(G, A)$ by obstructions—but an equally useful inter-