

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 24 (1978)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ORIGINS OF THE COHOMOLOGY OF GROUPS
Autor: Mac Lane, Saunders
Kapitel: 4. The Background in Abstract Algebra
DOI: <https://doi.org/10.5169/seals-49687>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 27.04.2026

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

4. THE BACKGROUND IN ABSTRACT ALGEBRA

Algebra treated by the use of axioms probably began in the late 19th century in the work of Dedekind, followed by E. H. Moore, Steinitz and others. However, modern or “abstract” algebra is a newer subject; it involves the use of axioms *and* conceptual methods to get a deeper understanding of disparate algebraic phenomena. As I have indicated elsewhere in a paper [1978] on the history of this subject, abstract algebra in this sense came into being in 1921, with a paper by Emmy Noether on “Ideal theorie in Ringbereichen”. This was the first paper in which “Ring” was used in its modern axiomatic sense, though the word “number ring” had been used in Hilbert’s 1898 *Zahlbericht*, while Fraenkel in 1916 had made partial attempts at axiomatics for rings. More important in this paper of Noether’s was the clear recognition that some arithmetic theorems known for special rings of algebraic integers or of polynomials could be formulated and proved better under general conditions—avoiding needless computational complexities and bringing out the conceptual structures involved.

This paper of Noether’s appears to have quickly stimulated many other studies in abstract algebra—both her own studies and those of her colleagues, collaborators, and pupils. In ten years, this provided the full background of abstract algebra, as formulated in van der Waerden’s book *Moderne Algebra* (Band I, 1930; Band II, 1931). The abstract spirit was clearly there, though some of the central notions do not yet have due emphasis. For example, the notion of a (left) module over a ring, so important for the cohomology of groups, came in a bit indirectly under the titles “linearformenmoduln” and “groups with operators”.

Related to our topic is the study of group extensions, which was stimulating by this emphasis on abstract algebra. The topic had come up before, at least implicitly, in I. Schur’s study of the projective representation of a group and hence of the multiplier. Group extension themselves were codified by Schreier [1926]. A group E with abelian normal subgroup A and quotient group G , in other words a short exact sequence

$$1 \rightarrow A \xrightarrow{i} E \xrightarrow{p} G \rightarrow 1, \quad (1)$$

is an *extension* of A by G . In such an extension, conjugation in E makes A a left $E/A = G$ module. If one chooses to each $x \in G$ a representative $u(x) \in E$ with $pu(x) = X$, the product of two such representatives has the form

$$u(x)u(y) = f(x, y)u(xy) \quad (2)$$

where the factor $f(x, y) \in A$ must satisfy an identity representing the associative law for the triple product $u(x)u(y)u(z)$; this f was called a “factor set”. The extension E is then completely determined by G , the G -module A , and this factor set f . It turns out from the associative law that f is exactly a two dimensional cocycle for G , and that the set of all extensions E of the given G module A by G is exactly the two dimensional cohomology group $H^2(G, A)$. Hence this group, as well as the one dimension cohomology group $H^1(G, A)$, was well known in the 1930’s. This made it possible for Eilenberg-Mac Lane and Eckmann to recognize in their papers cited above that the general cohomology of a group includes for dimension 2 the known case of group extensions.

In this description of group extensions by factor sets, the binary operation (of multiplication or perhaps addition) which makes $H^2(G, A)$ a group is given by the multiplication of two factor sets f, f' to form a new factor set

$$f(x, y)f'(x, y).$$

In his studies of group extension [1934], Baer had raised and answered the question of finding an invariant way of describing this multiplication of two extensions (1) and (1')—a description independent of the choice of representatives and now called the “Baer product” of extensions. He likewise had considered extensions of a *non-abelian* group A by a group G , and had observed that such an extension, realizing given operators of G on A , are not always possible. Indeed, there is a certain obstruction to forming such an extension, and this obstruction is a three-dimensional cohomology class of $H^3(G, Z)$ where Z is the center of A . This obstruction was identified in this way by Eilenberg-Mac Lane in 1947, and was a central element in the development of the cohomology of groups as an independent subject, not necessarily tied to the motivating topological examples.

5. THE BACKGROUND IN CLASS FIELD THEORY

In the early 20th century, linear algebra was an Anglo-American subject. Hamilton’s discovery of quaternions and C. S. Peirce’s utilization of idempotents had started the subject off. In 1905 Wedderburn had proved that any finite division algebra was commutative; one year later he proved his structure theorems. In a sense, they reduced the search for all finite