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Autor: Ahlfors, Lars V.
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8. AUTOMORPHIC FUNCTIONS AND BELTRAMI DIFFERENTIALS

Although this aspect has not been emphasized it should be clear that the author is trying to develop a theory which is immediately applicable to the study of discrete subgroups of G . All the definitions have been chosen with this in mind, and the relevant theorems for subgroups follow effortlessly.

Let G^0 be a discrete subgroup of G . A vector-valued function f is *automorphic* with respect to G^0 if $A^* f = f$, or more explicitly $A'(x)^{-1} f(Ax) = f(x)$ for all $A \in G^0$. Similarly, an SM_n -valued function v will be called a *Beltrami differential* for G^0 if $A^* v = v$, or $A'(x)^{-1} v(Ax) A'(x) = v(x)$, for all $A \in G^0$. If v is a Beltrami differential, then $A^*(\rho v dx) = \rho v dx$ for all $A \in G^0$, and $\rho v dx$ is called an n th order differential. The terminology is borrowed from the corresponding notions for $n = 2$.

If v is Beltrami and in L^∞ , then it is also in $L^p(B)$ for all p , and Theorems 2-5 are applicable. They gain added significance from the fact that Iv is automatically automorphic with respect to G^0 (it is easy to show that $A^* Iv = IA^* v$ for all v and $A \in G$). As a consequence SIv is Beltrami, and by Theorem 2 the same is true of Γv . It follows that Theorems 2-5 may be interpreted as referring to the quotient space $G^0 \backslash B$, provided that we start from the hypothesis $v \in L^\infty$. In the conclusion we know, for instance, that

$$\int_B \|SI \gamma\|^p dx = \int_{G^0 \backslash B} \|SI v\|^p \rho_0 dx < \infty$$

where, by a theorem of Godement,

$$\rho_0(x) = \sum_{A \in G^0} |A'(x)|^n$$

is known to converge.

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