

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 24 (1978)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: SINGULAR INTEGRAL EQUATION CONNECTED WITH QUASICONFORMAL MAPPINGS IN SPACE
Autor: Ahlfors, Lars V.
Kapitel: 8. Automorphic functions and beltrami differentials
DOI: <https://doi.org/10.5169/seals-49703>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 19.08.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

8. AUTOMORPHIC FUNCTIONS AND BELTRAMI DIFFERENTIALS

Although this aspect has not been emphasized it should be clear that the author is trying to develop a theory which is immediately applicable to the study of discrete subgroups of G . All the definitions have been chosen with this in mind, and the relevant theorems for subgroups follow effortlessly.

Let G^0 be a discrete subgroup of G . A vector-valued function f is *automorphic* with respect to G^0 if $A^*f = f$, or more explicitly $A'(x)^{-1}f(Ax) = f(x)$ for all $A \in G^0$. Similarly, an SM_n -valued function v will be called a *Beltrami differential* for G^0 if $A^*v = v$, or $A'(x)^{-1}v(Ax)A'(x) = v(x)$, for all $A \in G^0$. If v is a Beltrami differential, then $A^*(\rho v dx) = \rho v dx$ for all $A \in G^0$, and $\rho v dx$ is called an n th order differential. The terminology is borrowed from the corresponding notions for $n = 2$.

If v is Beltrami and in L^∞ , then it is also in $L^p(B)$ for all p , and Theorems 2-5 are applicable. They gain added significance from the fact that Iv is automatically automorphic with respect to G^0 (it is easy to show that $A^*Iv = IA^*v$ for all v and $A \in G$). As a consequence SIv is Beltrami, and by Theorem 2 the same is true of Γv . It follows that Theorems 2-5 may be interpreted as referring to the quotient space $G^0 \backslash B$, provided that we start from the hypothesis $v \in L^\infty$. In the conclusion we know, for instance, that

$$\int_B || SIv ||^p dx = \int_{G^0 \backslash B} || SIv ||^p \rho_0 dx < \infty$$

where, by a theorem of Godement,

$$\rho_0(x) = \sum_{A \in G^0} |A'(x)|^n$$

is known to converge.

REFERENCES

- [1] AHLFORS, Lars V. Kleinsche Gruppen in der Ebene und im Raum. *Festband zum 70. Geburtstag von Prof. Rolf Nevanlinna*, Springer Verlag, Berlin, New York, 1966, p. 7-15.
- [2] —— Hyperbolic Motions. *Nagoya Math. J.* 29 (1967), pp. 163-166.
- [3] —— Conditions for Quasiconformal Deformations in Several Variables. *Contributions to Analysis. A Collection of Papers Dedicated to Lipman Bers*, pp. 19-25, Academic Press, New York and London, 1974.
- [4] —— Invariant Operators and Integral Representations in Hyperbolic Space. *Math. Scand.* 36 (1975), pp. 27-43.