A SINGULAR INTEGRAL EQUATION CONNECTED WITH QUASICONFORMAL MAPPINGS IN SPACE

by Lars V. AHLFORS [1])

Dedicated to Albert Pfluger for his seventieth birthday

1. INTRODUCTION

This paper continues the author's investigation of two differential operators, S and S^*, which arise naturally in the study of infinitesimal quasiconformal mappings in n dimensions (see References). If Ω is open in $\mathbf{R}^n$ the operator S acts on functions $f : \Omega \to \mathbf{R}^n$ and has values $Sf \in SM_n$ where SM_n is the space of symmetric $n \times n$ matrices with zero trace. Definitions are in Sec. 2.

A key question is the solvability of the inhomogeneous equation $Sf = v$. For $n = 2$, Sf can be identified with the complex derivative $f_{\bar{z}}$ of a complex-valued function, and the problem is that of recovering f from $f_{\bar{z}}$. As well known, this problem has always a solution, and it is given by the generalized Cauchy formula, also known as Pompeiu's formula. For $n > 2$ the right hand member v, an SM_n-valued function, must satisfy certain conditions, which are known in principle, as limiting cases of the Weyl-Schouten conditions of vanishing conformal curvature.

These conditions, although explicit, are quite intractable. It is therefore rather surprising that a necessary and sufficient condition for $Sf = v$ to be solvable can be expressed as a singular homogeneous integral equation satisfied by v. This integral equation can be treated by the methods of Calderon and Zygmund.

2. DEFINITIONS AND NOTATIONS

A quasiconformal homeomorphism $F : \Omega \to F(\Omega)$ is known to be differentiable almost everywhere. We denote its Jacobian matrix by DF. The normalized Jacobian is $XF = (\det DF)^{-1/n} DF$, and $MF = {}^tXF \cdot XF$

[1]) Supported by NSF Grant GP-38886.