

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 24 (1978)  
**Heft:** 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** SINGULAR INTEGRAL EQUATION CONNECTED WITH  
QUASICONFORMAL MAPPINGS IN SPACE

**Autor:** Ahlfors, Lars V.  
**Kapitel:** 1. Introduction  
**DOI:** <https://doi.org/10.5169/seals-49703>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 27.04.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

# A SINGULAR INTEGRAL EQUATION CONNECTED WITH QUASICONFORMAL MAPPINGS IN SPACE

by Lars V. AHLFORS <sup>1)</sup>

*Dedicated to Albert Pfluger for his seventieth birthday*

## 1. INTRODUCTION

This paper continues the author's investigation of two differential operators,  $S$  and  $S^*$ , which arise naturally in the study of infinitesimal quasiconformal mappings in  $n$  dimensions (see References). If  $\Omega$  is open in  $\mathbf{R}^n$  the operator  $S$  acts on functions  $f : \Omega \rightarrow \mathbf{R}^n$  and has values  $Sf \in SM_n$ , where  $SM_n$  is the space of symmetric  $n \times n$  matrices with zero trace. Definitions are in Sec. 2.

A key question is the solvability of the inhomogeneous equation  $Sf = v$ . For  $n = 2$ ,  $Sf$  can be identified with the complex derivative  $f_{\bar{z}}$  of a complex-valued function, and the problem is that of recovering  $f$  from  $f_{\bar{z}}$ . As well known, this problem has always a solution, and it is given by the generalized Cauchy formula, also known as Pompeiu's formula. For  $n > 2$  the right hand member  $v$ , an  $SM_n$ -valued function, must satisfy certain conditions, which are known in principle, as limiting cases of the Weyl-Schouten conditions of vanishing conformal curvature.

These conditions, although explicit, are quite intractable. It is therefore rather surprising that a necessary and sufficient condition for  $Sf = v$  to be solvable can be expressed as a singular homogeneous integral equation satisfied by  $v$ . This integral equation can be treated by the methods of Calderon and Zygmund.

## 2. DEFINITIONS AND NOTATIONS

A quasiconformal homeomorphism  $F : \Omega \rightarrow F(\Omega)$  is known to be differentiable almost everywhere. We denote its Jacobian matrix by  $DF$ . The normalized Jacobian is  $XF = (\det DF)^{-1/n} DF$ , and  $MF = {}^tXF \cdot XF$

---

<sup>1)</sup> Supported by NSF Grant GP-38886.