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# ORIGINS OF THE COHOMOLOGY OF GROUPS<sup>1</sup>

by Saunders MAC LANE

## 1. THE HISTORICAL QUESTIONS

This paper is a small essay on the history of contemporary mathematics. It will examine the questions: What were the origins of the theory of the cohomology of groups? What were the essential steps in the development of this theory? What were the effects of this development in related fields of mathematics? These questions seem appropriate to a conference in Zurich, because major steps in the development of this subject took place here at the Eidgenössische Technische Hochschule. These questions may also be typical of questions that can be put about the development of other parts of mathematics in the twentieth century. Here are some of these questions: How does the interchange of ideas between different fields of mathematics come about? Which ideas (or, which research papers) are of essential novelty or originality and which are derivative? Do some ideas arrive before their time, and so are neglected? What are the differences between mathematical developments seen beforehand, or seen after the fact—and is there not a third perspective, that of mathematical ideas as they are in process of development?

## 2. FUNDAMENTAL GROUP AND 2ND BETTI GROUP

On September 12, 1941 Heinz Hopf communicated to the *Commentarii Mathematici Helvetici* his paper “Fundamentalgruppe und zweite Bettische Gruppe”. This paper proved the

**THEOREM.** Each group  $G$  determines, by an algebraic process, a group  $G_1^*$  which is not generally zero. If  $G$  is the fundamental group of a complex  $K$  with second Betti group  $B^2 = H_2(K, \mathbf{Z})$ , and if  $S^2$  is the spherical subgroup of  $B^2$ , then

$$B^2/S^2 \cong G_1^* \tag{1}$$

<sup>1)</sup> Presented at the Colloquium on Topology and Algebra, April 1977, Zurich.