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classes by this means, see [10] and [9]. On the other hand no “stable class” has ever been detected by a homogenous example.

Using the Lefschetz-Hyperplane Theorem for  $G^C/P^C$ , Haefliger and I have quite recently been able to explain this. Indeed one has the following quite general fact:

THEOREM. *All stable classes of  $F\Gamma$  in  $H^*(|G/P \cdot \Gamma|)$  vanish.*

Here  $P$  is parabolic in  $G$ , and  $\Gamma \subset G$ . Presumably the same is true in all homogenous cases but this is not clear to us so far.

The master of non homogenous actions has been first and foremost Thurston, and last year Heitch has been able to considerably extend Thurston's computations [9]. I will not be able to report on this work here, except to state what Thurston's constructions imply for the smooth cohomology of  $\text{Diff}(S^1)$ . He shows first of all that  $e$  and  $\omega$  are independent, and in fact he constructs a smooth family of actions of the group

$$\Pi = \{X, Y, U, V \mid [X, Y] = [U, V]\}$$

on  $S^1$  for which the class  $\omega$  varies continuously. See [2] for details in this context.) He also constructed examples to show that  $\omega^n \neq 0$ . On the other hand we still have no example for which  $e^n$ ,  $n \geq 2$  is nonzero.

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