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# REPRESENTATION OF COMPLETELY CONVEX FUNCTIONS BY THE EXTREME-POINT METHOD

by CHRISTIAN BERG

## 0. INTRODUCTION

A function  $f : ]0, 1[ \rightarrow \mathbf{R}$  is called completely convex, if it is  $C^\infty$  and  $(-1)^k f^{(2k)} \geq 0$  for all  $k \geq 0$ . A completely convex function  $f$  is called minimal if  $f(x) - a \sin(\pi x)$  is not completely convex for any number  $a > 0$ . Widder showed (cf. [5]) that a completely convex function can be extended to an entire holomorphic function, and in the paper [6] he proved that a minimal completely convex function can be expanded in a Lidstone series. This indicates that the Lidstone polynomials lie on the extreme rays of the cone  $W$  of completely convex functions.

The purpose of the present paper is to treat the completely convex functions by the extreme-point method and to obtain the expansion in Lidstone series as a special case of the Choquet representation theorem.

We will proceed as follows: In the topology of point-wise convergence the set  $W$  of completely convex functions is a closed, metrizable convex cone. We prove directly that the extreme rays of  $W$  are generated by certain polynomials — essentially the Lidstone polynomials — and the function  $\sin(\pi x)$ . The occurrence of the extreme ray generated by  $\sin(\pi x)$  is related to the fact that only minimal completely convex functions can be expanded in Lidstone series.

The cone  $W$  has a compact convex base  $B$ , and the extreme points of  $B$  are determined. It turns out that  $B$  is a Bauer simplex, i.e.  $B$  is a simplex and the extreme points form a closed set.

The author wants to acknowledge Widder's paper [6] as a source of inspiration. The reason for writing this paper is that we felt it natural to investigate the cone  $W$  by the extreme-point method.

Recently Mugler [2] showed that real part of the holomorphic extension of  $f \in W$  to the strip  $\operatorname{Re} z \in ]0, 1[$  is completely convex on each segment  $\{x + iy \mid 0 < x < 1\}$ . We give a very short proof of this result.