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Autor: Doran, Robert S. / Wichmann, Josef

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tion is the Gelfand-Naimark-Segal construction discussed earlier in this article.

A substantial literature has now emerged from this new algebraic point of view and a recent book by G. Emch [18] has been written with the express purpose of offering a systematic introduction to the ideas and techniques of the C^* -algebra approach to physical problems. The authors recommend this book to the reader who would like to pursue this subject further. The book contains a bibliography of more than four hundred items which should aid the interested reader in his study of this new and interesting application of operator algebras.

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Robert S. Doran
Josef Wichmann

Texas Christian University
Department of Mathematics
Fort Worth, Texas 76129
USA