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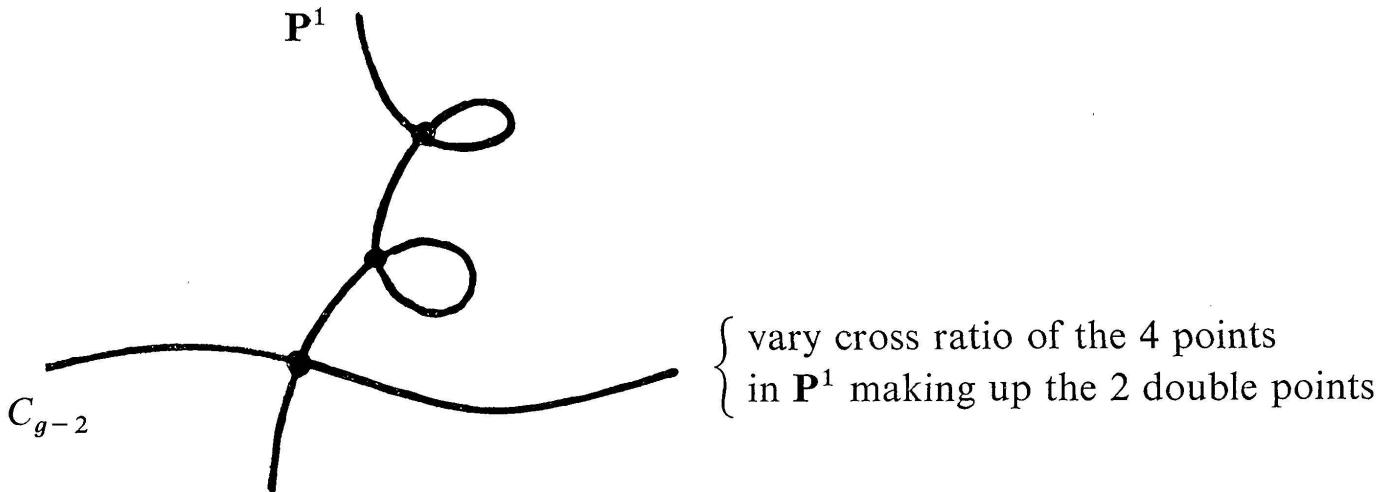
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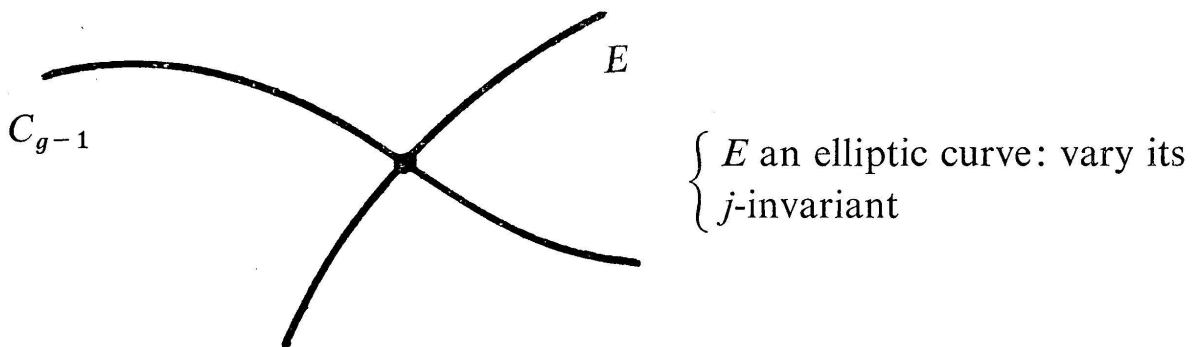
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where C_{g-2} is a fixed genus $(g-2)$ component, then $\lambda|_{S_1} = \mathcal{O}_{S_1}$, hence sections of λ always collapse such families.

(2) If S_2 is a curve in $\overline{\mathcal{M}}_g$ composed of curves of the form:



where C_{g-1} is a fixed genus $(g-1)$ component, then $\lambda^{11} \otimes \delta^{-1}|_{S_2} = \mathcal{O}_{S_2}$ i.e. $\lambda^{11} \otimes \delta^{-1}$ collapses these families.

We omit the details.

APPENDIX

We wish to fill in the gap in the proof of Proposition 5.5 on page 95. The difficulty occurs if the support of \mathcal{S} , i.e. $(0) \times L_1$, contains some of the components of C_2 meeting C_1 . In this case, the inequality

$$e_L(\mathcal{S}_2) \geq w$$

is not clear. Indeed, if D_1, \dots, D_k are the components of C_2 meeting C_1 , $w_i = \#(D_i \cap C_1)$, and \mathcal{K}_i is the pull-back of \mathcal{S}_2 to D_i , then

$$\begin{aligned} e_L(\mathcal{F}_2) &= \sum e_L(\mathcal{K}_i) \\ e_L(\mathcal{K}_i) &\geq w_i \text{ if } L_1 \not\cong D_i \\ &= 2 \deg D_i \text{ if } L_1 \cong D_i. \end{aligned}$$

Now suppose C_1 is *irreducible* and $D_i \subseteq L_1$. Then (5.7) is modified to:

$$2 \deg D_i + 2 \deg C_1 \leq \frac{16}{7} (n_1 + 1).$$

Since C_1 spans L_1 , $n_1 \leq \deg C_1$. Substituting this, we find

$$\deg D_i \leq \frac{\deg C_1}{7} + \frac{8}{7}$$

hence $\deg D_i \leq \deg C_1$ (except in the lowest case $\deg C_1 = 1$; in this case, C_1 is a line, so $C_1 = L_1$ and $\text{Supp } \mathcal{K}_i = D_i \cap L_1 \not\subseteq D_i$). Now the reverse of this inequality cannot be true too. This means that if we apply the same argument to

$$C_{\text{red}} = D_i \cup \overline{(C - D_i)}$$

then the linear span M of D_i cannot contain C_1 . Therefore

$$w_i + 2 \deg D_i \leq \frac{16}{7} (\dim M_i + 1) \leq \frac{16}{7} (\deg D_i + 1)$$

$$\begin{aligned} \therefore w_i &\leq 2 \deg D_i \\ \therefore e_L(\mathcal{K}_i) &\geq w_i \text{ in all cases} \\ \therefore e_L(\mathcal{F}_2) &\geq w \text{ as required.} \end{aligned}$$

This proves (5.7) if C_1 is irreducible, hence (a) and (b) that follow are correct. In particular, (b) shows that $\mathcal{O}_{C_1}(1)(-\mathcal{W})$ always has sections, unless C_1 is a line and $\# \mathcal{W} = 2$. The next paragraph shows that C is embedded by a complete linear system. So when $\Gamma(\mathcal{O}_{C_1}(1)(-\mathcal{W})) \neq (0)$, there is a hyperplane containing all components of C except C_1 . Returning to the general case of (5.7) where C_1 is any subset of the components of C , it follows that the linear span L_1 of C_1 contains only C_1 and the *lines* D_i which meet C_1 in 2 points. For these, $\#(D_i \cap C_1) = 2 \deg D_i$, so in all cases it is true that $e_L(\mathcal{F}_2) \geq w$ as required.