cannot have more than $c' \sigma \log \sigma$ zeros in a disc $|z| \leqslant c''$, where c', c'' depend only on p. We are indebted for the above details to D. W. Masser (for a problem involving zeros of polynomials in several variables see his [13]).

To extend our method to a class of functions wider than that given by (44) is practical provided only that one can usefully estimate the determinants arising in lemma 2. This can certainly be done in the case

$$F(z) = \sum_{h=1}^{m} \sum_{t=1}^{\rho(h)} a_{ht} (\log z)^{t-1} z^{\alpha_h},$$

for details see van der Poorten [22]. A similar argument should allow one to deal with functions

$$\sum_{h=1}^{\sigma} b_h f_{\mu_h}(z),$$

where f_μ is given by (45); now lemma 5 allows one to consider rather surprising functions. There are further, rather isolated cases where one can deal with the determinants; for some examples, and further references see [21].

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