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ON THE NUMBER OF ZEROS OF FUNCTIONS

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0. Introduction

The object of this note is to give a complete description of a technique that leads to estimates for the number of zeros (always assumed to be counted according to multiplicity) of certain classes of functions in discs of given radius and centre in the complex plane. As we show, the technique also suffices to prove that functions cannot be (relatively) small too often in discs. In order that this paper may be a useful source we have made our proofs essentially self-contained; our lemmas are often more general than is required for the immediate applications and we have taken the opportunity to mention various formulae and tricks which, though no doubt well-known in the folklore, are by no means readily accessible in the literature.

We principally consider the case of exponential polynomials, that is, solutions of homogeneous linear differential equations with constant coefficients, and then briefly indicate the manner in which the method described extends to a very much wider class of functions.

Though the results are of general interest, the principal motive for their formulation has resided in their application in the theory of transcendental numbers. In this context one constructs auxiliary functions and shows that the contradiction of the result to be proved implies that, contrary to the construction, the auxiliary function vanishes identically; see, Gelfond [7], Chapter III, Tijdeman [28], Brownawell [2], Waldschmidt [34], Cudnovskii [3] (see Waldschmidt [35] for a summary) for typical application of theorem 1. The second result, theorem 2 is important in obtaining transcendence measures as well as in recent work on algebraic independence; for a recent application see, for example Cijsouw [4].

The present theory would seem to have been initiated by the work of Gelfond, see [7], p. 140ff. The work of Tijdeman [26], see also [27], pıovided the major breakthrough which has simplified subsequent results.

There is also an analogous p-adic theory, see for example Shorey [25]. In fact the results are simpler in the p-adic case as can be seen in the recent work of van der Poorten [24], see also [23].