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$$\psi(\alpha_0)\psi(q) = \psi(m_0/f)\psi(n)$$

or

$$\bar{\psi}(n)\psi(q) = \bar{\psi}(\alpha_0)\psi\left(\frac{m_0}{f}\right) \tag{16}$$

Multiplying (14), (15) and (16) we find $K(\alpha) = H(\alpha)$.

4. SPECIAL CASES

(a) Theorem *B* implies that $\tau(\chi) \neq 0$ if and only if $R = 1$, that is, if and only if m/f is squarefree and has no common divisor with f . We have then

$$\tau(\chi) = \mu\left(\frac{m}{f}\right)\psi\left(\frac{m}{f}\right)\tau(\psi) \tag{17}$$

and

$$\mathcal{G}(\alpha, \chi) = g(\alpha)\tau(\chi) \tag{18}$$

On the other hand, if m/f is not square free or has a common divisor with f , then the right hand side of (17) is zero. So, (17) holds for any character χ . For another proof of this see [4], p. 148.

(b) If $\chi = \chi_0 =$ principal character mod m , then $f = 1$, $\psi \equiv 1$, $\tau(\psi) = 1$, $q = \tilde{m} =$ squarefree kernel of m , $R = m/\tilde{m}$, and $\mathcal{G}(\alpha, \chi_0) = C_m(\alpha) =$ RAMANUJANS SUM.

Theorem *B* gives the well-known formula:

$$C_m(\alpha) = 0 \quad \text{if} \quad \frac{m}{\tilde{m}} \nmid \alpha$$

$$C_m\left(\frac{m}{\tilde{m}}n\right) = \frac{m}{\tilde{m}}\mu(\tilde{m})\mu((n, \tilde{m}))\varphi((n, \tilde{m})).$$

From (17) we get for all m

$$C_m(1) = \mu(m).$$

5. Remarks: (a) It is clear that $\mathcal{G}(\alpha, \chi)$ cannot vanish identically. So by 4. (a), formula (1) can only hold if $R = 1$, and if $g(\alpha) = \bar{\chi}(\alpha)$ for all α . But this is only possible if $q = 1$, i.e. if $m = f$. This shows that (1) characterises primitive characters, a fact proved by T.M. Apostol [5].

(b) The last named author also proved ([6]), that if the functional equation (4) holds, then χ is primitive. One may prove this by comparing (4) and (5), which gives $\mathcal{G}(\alpha, \chi) = \bar{\chi}(\alpha) \tau(\chi)$; this in turn implies that χ is primitive, by the former remark. Still another proof is as follows: If $q = 1$, then $L(s, \chi) = L(s, \psi)$ and $L(s, \bar{\chi}) = L(s, \bar{\psi})$. So if (4) holds, we get $m = f$, hence χ is primitive. If $q > 1$, then $L(s, \chi)$ must have nonreal zeroes on the imaginary axis; hence if (4) holds, $L(s, \bar{\chi})$ has zeroes on the line $\operatorname{Re} s = 1$, contradicting a well known theorem on L -series.

REFERENCES

- [1] HASSE, H. *Vorlesungen über Zahlentheorie*. 2. ed. Springer (1964), pp. 444-450.
- [2] MONTGOMERY, H. L. and R. C. VAUGHAN. The exceptional set in Goldbach's problem. *Acta Arith.* XXVII (1975), pp. 353-370.
- [3] BERNDT, B. C. and L. SCHOENFELD. Periodic analogues of the Euler-Maclaurin and Poisson summation formulas with applications to number theory. *Acta Arith.* XXVIII (1975), pp. 23-68.
- [4] DAVENPORT, H. *Multiplicative Number Theory*. Chicago (1967).
- [5] APOSTOL, T. M. Euler's φ -function and separable Gauss sums. *Proc. AMS* 24 (1970), pp. 482-485.
- [6] ——— Dirichlet L -functions and Dirichlet characters, *Proc. AMS* 31 (1972), pp. 384-386.
- [7] CHANDRASEKHARAN, K. *Arithmetical functions*. Springer (1970), pp. 146-154.

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