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$$\tilde{F} - F = \sum_{\alpha; \beta_1, \dots, \beta_\alpha} p_{\beta_1, \dots, \beta_\alpha}(x_1, \dots, x_n) \\ \times \varphi_{\beta_1, \dots, \beta_\alpha}(q_{\beta_1, \dots, \beta_\alpha, 1}(x_1, \dots, x_n), \dots, q_{\beta_1, \dots, \beta_\alpha, k}(x_1, \dots, x_n)) + R(x_1, \dots, x_n),$$

where $|R(x_1, \dots, x_n)| \leq \gamma(\varepsilon)\varepsilon$, $\gamma(\varepsilon) \rightarrow 0$ as $\varepsilon \rightarrow 0$, and

$$\varepsilon = \max_{\alpha; \beta_1, \dots, \beta_\alpha} \sup_t |\varphi_{\beta_1, \dots, \beta_\alpha}(t_1, \dots, t_k)| \\ \leq \lambda \sup_{x \in G_n} |\tilde{F}(x_1, \dots, x_n) - F(x_1, \dots, x_n)|.$$

That $\gamma(\varepsilon) \rightarrow 0$ as $\varepsilon \rightarrow 0$ follows from the fact that as $\varepsilon \rightarrow 0$ the quantity

$$\varepsilon' = \max_{\alpha; \beta_1, \dots, \beta_\alpha} \sum_{i=1}^k \sup \left| \frac{\partial \varphi_{\beta_1, \dots, \beta_\alpha}(t_1, \dots, t_k)}{\partial t_i} \right| \rightarrow 0,$$

provided only that the modulus of continuity of the partial derivatives of the functions $\{\varphi_{\beta_1, \dots, \beta_\alpha}(t_1, \dots, t_k)\}$ is fixed. By 5.1.10 it follows that $r(A, F) \leq k$ in some subregion $G_n \subset D_n$. So we have obtained a contradiction to the assumption that $r(A, F) > k$ in any subregion $G_n \subset D_n$ and this proves the theorem.

REFERENCES

- [1] HILBERT, D. Mathematische Probleme. *Nachr. Akad. Wiss. Göttingen* (1900), 253-297;
Gesammelte Abhandlungen, Bd. 3 (1935), 290-329.
- [2] OSTROWSKI, A. Über Dirichletsche Reihen und algebraische Differentialgleichungen.
Math. Z. 8 (1920), 241-298.
- [3] HILBERT, D. Über die Gleichung neunten Grades. *Math. Ann.* 97 (1927), 243-250;
Gesammelte Abhandlungen, Bd. 2 (1933), 393-400.
- [4] VITUSHKIN, A. G. On Hilbert's thirteenth problem. *Dokl. Akad. Nauk SSSR* 95 (1954), 701-704.
- [5] BIEBERBACH, L. Bemerkung zum dreizehnten Hilbertschen Problem. *J. Reine Angew. Math.* 165 (1931), 89-92.
- [6] —— Einfluss von Hilberts Pariser Vortrag über „Mathematische Probleme“. *Naturwissenschaften* 51 (1930), 1101-1111.
- [7] KOLMOGOROV, A. N. On the representation of continuous functions of several variables by superpositions of continuous functions of fewer variables. *Dokl. Akad. Nauk SSSR* 108 (1956), 179-182. *Amer. Math. Soc. Transl.* (2) 17 (1961), 369-373.
- [8] ARNOL'D, V. I. On functions of three variables. *Dokl. Akad. Nauk SSSR* 114 (1957), 679-681.
- [9] KOLMOGOROV, A. N. On the representation of continuous functions of several variables by superpositions of continuous functions of one variable and addition. *Dokl. Akad. Nauk SSSR* 114 (1957), 953-956. *Amer. Math. Soc. Transl.* (2) 28 (1963), 55-59.

- [10] VITUSHKIN, A. G. and G. M. HENKIN. Linear superpositions of functions. *Uspehi Mat. Nauk* 22, N 1 (1967), 77-124.
- [11] VITUSHKIN, A. G. On 13-th Problem of Hilbert. *Hilbert's problem*, Izdat. Nauka, 1969.
- [12] LORENTZ, G. G. On the 13-th problem of Hilbert. "Mathematical Developments Arising from the Hilbert Problems", Proceedings of Symposia in Pure Mathematics, Vol. 28 (De Kalb, Illinois, May 1974), American Mathematical Society Providence, RI, 1975.
- [13] WIMAN, A. Über die Anwendung der Tschirnhausen-Transformation auf die Reduktion algebraischer Gleichungen. *Nova Acta Soc. Sci. Upsal.* (1928), 3-8.
- [14] CHEBOTAREV, G. N. On the problem of the resolvent. *Kazan. Gos. Univ. Uch. Zap.* 114 (1954), 189-193.
- [15] —— On certain questions of the problem of resolvents. *Sobranie sochinenii* (collected works), Vol. 1 (1949), 255-340.
- [16] MOROZOV, V. V. On certain questions of the problem of resolvents. *Kazan. Gos. Univ. Uch. Zap.* 114 (1954), 173-187.
- [17] LIN, V. Superpositions of algebraic functions. *Functional analysis and its applications* 10, N 1 (1976), 37-45.
- [18] ARNOL'D, V. I. Topological invariants of algebraic functions, *Functional analysis and its applications* 4, N 2 (1970), 1-9.
- [19] HOVANSKI, A. G. On superpositions of holomorphic functions with radicals. *Uspehi Mat. Nauk* 26, N 2 (1971).
- [20] —— On a representation of functions by quadrature. *Uspehi Mat. Nauk* 26, N 4 (1971), 251-252.
- [21] VITUSHKIN, A. G. *O mnogomernykh variatsiyakh* (On multidimensional variations), Gostehizdat, Moscow 1955.
- [22] IVANOV, L. D. *Variatsii mnogestv i funktsii* (Variations of sets and functions). Izdat. Nauka, Moscow 1975.
- [23] KOLMOGOROV, A. N. Estimates of the minimal number of elements of ϵ -nets in various functional classes and their application of representability of functions of several variables by superpositions of functions of fewer variables. *Uspehi Mat. Nauk* 10, N 1 (1955), 192-193; *Dokl. Akad. Nauk SSSR* 101 (1955), 192-194.
- [24] LORENTZ, G. G. Lower bounds for the degree of approximation. *Trans. Amer. Math. Soc.* 97, N 1 (1960), 25-34.
- [25] SHAPIRO, H. S. Some negative theorems of approximation theory. *Michigan Math. J.* 11, N 3 (1964), 211-217.
- [26] EROKHIN, V. D. On the test approximation of analytic functions by rational functions with free poles. *Dokl. Akad. Nauk SSSR* 128 (1959), 29-32.
- [27] TIHOMIROV, V. M. Diameters of sets in functional spaces and the theory of test approximations. *Uspehi Mat. Nauk* 15, N 3 (1960), 81-120. *Russian Math. Surveys* 15, N 3 (1960), 75-111.
- [28] DUNFORD, N. and J. T. SCHWARTZ. *Linear Operators I, General Theory*. Interscience Publishers, New York and London, 1958. Translation: *Lineinyye operatory, Obshchaya teoriya*, Izdat. Inost. Lit. Moscow 1962, 526-527.
- [29] KRONROD, A. S. On functions of two variables. *Uspehi Mat. Nauk* 5, N 1 (1950), 24-134.
- [30] MENGER, K. *Kurventheorie*. Berlin-Leipzig, 1932, Chap. X.
- [31] ARNOL'D, V. I. On the representation of continuous functions of three variables by superpositions of continuous functions of two variables. *Math. Sb.* 48 (90) (1959), 3-74; 56 (98) (1962, 392). *Amer. Math. Soc. Transl. (2)* 28 (1963), 61-147.

- [32] BARI, N. Mémoire sur la représentation finie des fonctions continues. *Math. Ann.* 103 (1930), 145-248, 598-653.
- [33] ARNOL'D, V. I. On the representability of functions of several variables in the form of superpositions of functions of fewer variables. *Matem. Prosv.* 3 (1958).
- [34] LORENTZ, G. G. Metric entropy, widths and superpositions of functions. *Amer. Math. Monthly* 69 (1962), 469-485.
- [35] TIHOMIROV, V. M. Kolmogorov's work on the ε -entropy of functional classes and superpositions of functions. *Uspehi Mat. Nauk* 18, N 5 (1963), 55-92. *Russian Mathematical Surveys* 18, N 5 (1963), 51-87.
- [36] KAHANE, J.-P. Sur le théorème de superposition de Kolmogorov. *Journal of Approximation theory* 13, N 3 (1975), 229-234.
- [37] FRIDMAN, B. Improvement in the smoothness of functions in the Kolmogorov superposition theorem. *Dokl. Akad. Nauk SSSR* 177 (1967), 5.
- [38] DOSS, R. On the representation of the continuous of two variables by means of addition and continuous functions of one variable. *Colloq. Math.* 10 (1963), 249-259.
- [39] BASSALYGO, L. A. On the representation of continuous functions of two variables by means of continuous functions of one variable. *Vestnik Moskov. Univ. Ser. I, Mat. Meh.* 21 (1966), 58-63.
- [40] SPRECHER, D. A. On the structure of continuous functions of several variables. *Trans. Amer. Math. Soc.* 115 (1965), 340-355.
- [41] ARNOL'D, V. I. On the representability of functions of two variables in the form $\kappa(\varphi(x) + \psi(y))$. *Uspehi Mat. Nauk* 12, N 2 (1957), 119-121.
- [42] OFMAN, Yu. P. On the best approximation of functions of two variables by functions of the form $\varphi(x) + \psi(y)$. *Izv. Akad. Nauk SSSR Ser. Mat.* 25 (1961), 239-252.
- [43] MOTORNYI, V. P. On the question of the best approximation of functions of two variables by functions of the form $\varphi(x) + \psi(y)$. *Izv. Akad. Nauk SSSR Ser. Mat.* 27 (1963), 1211-1214.
- [44] SPRECHER, D. A. On the structure of representations of continuous functions of several variables as finite sums of continuous functions of one variable. *Proc. Amer. Math. Soc.* 17 (1966), 98-105.
- [45] HENKIN, G. M. On linear superpositions of continuously differentiable functions. *Dokl. Akad. Nauk SSSR* 157 (1964), 288-290. *Soviet Math. Dokl.* 5 (1964), 948-950.
- [46] FRIDMAN, B. The set of linear superpositions is nowhere dense in the space of continuous functions of several variables. *Izv. Akad. Nauk SSSR Ser. Mat.* 36, N 4 (1972), 814-846.
- [47] VITUSHKIN, A. G. Some properties of linear superpositions of smooth functions. *Dokl. Akad. Nauk SSSR* 156 (1964), 1003-1006. *Soviet Math. Dokl.* 5 (1964), 741-744.
- [48] — Proof of the existence of analytic functions of several variables not representable by linear superpositions of continuously differentiable functions of fewer variables. *Dokl. Akad. Nauk SSSR* 156 (1964), 1258-1261. *Soviet Math. Dokl.* 5 (1964), 793-796.
- [49] — *Otsenka slozhnosti zadachi tabulirovaniya* (Estimation of the complexity of the tabulation problem). Fizmatgiz, Moscow 1959. *Theory of the transmission and processing of information*. Pergamon press, 1961.
- [50] KOLMOGOROV, A. N. and V. M. TIHOMIROV. ε -entropy and ε -capacity of sets in function spaces. *Uspehi Mat. Nauk* 14, N 2 (1959), 3-86.
- [51] BUSLAEV, V. I. and A. G. VITUSHKIN. An estimate of the code length of signals with finite spectrum in connection with sound recording problems. *Izv. Akad. Nauk SSSR Ser. Mat.* 38, N 4 (1974), 867-895.

- [52] VITUSHKIN, A. G. Coding of signals with finite spectrum and sound recording problems. *Proceedings of the International Congress of Mathematicians*, Vancouver, 1974.
- [53] SMUSHKO, V. V. Entropy of the class of entire functions with the frequency dependent metric. *Izv. Akad. Nauk SSSR Ser. Mat.* 40, N 5 (1976), 1173-1186.
- [54] HEDBERG, T. Sur les réarrangements de fonctions de la classe A et les ensembles d'interpolation pour $A(D^2)$. *C. R. Acad. Sc. Paris* 270 (1970), 1491-1494.

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