

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 23 (1977)  
**Heft:** 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** ON REPRESENTATION OF FUNCTIONS BY MEANS OF SUPERPOSITIONS AND RELATED TOPICS  
**Autor:** Vitushkin, A. G.

**Bibliographie**  
**DOI:** <https://doi.org/10.5169/seals-48931>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 29.04.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

$$\begin{aligned} \tilde{F} - F &= \sum_{\alpha; \beta_1, \dots, \beta_\alpha} p_{\beta_1, \dots, \beta_\alpha}(x_1, \dots, x_n) \\ &\times \varphi_{\beta_1, \dots, \beta_\alpha}(q_{\beta_1, \dots, \beta_\alpha, 1}(x_1, \dots, x_n), \dots, q_{\beta_1, \dots, \beta_\alpha, k}(x_1, \dots, x_n)) + R(x_1, \dots, x_n), \end{aligned}$$

where  $|R(x_1, \dots, x_n)| \leq \gamma(\varepsilon)\varepsilon$ ,  $\gamma(\varepsilon) \rightarrow 0$  as  $\varepsilon \rightarrow 0$ , and

$$\begin{aligned} \varepsilon &= \max_{\alpha; \beta_1, \dots, \beta_\alpha} \sup_t |\varphi_{\beta_1, \dots, \beta_\alpha}(t_1, \dots, t_k)| \\ &\leq \lambda \sup_{x \in G_n} |\tilde{F}(x_1, \dots, x_n) - F(x_1, \dots, x_n)|. \end{aligned}$$

That  $\gamma(\varepsilon) \rightarrow 0$  as  $\varepsilon \rightarrow 0$  follows from the fact that as  $\varepsilon \rightarrow 0$  the quantity

$$\varepsilon' = \max_{\alpha; \beta_1, \dots, \beta_\alpha} \sum_{i=1}^k \sup \left| \frac{\partial \varphi_{\beta_1, \dots, \beta_\alpha}(t_1, \dots, t_k)}{\partial t_i} \right| \rightarrow 0,$$

provided only that the modulus of continuity of the partial derivatives of the functions  $\{\varphi_{\beta_1, \dots, \beta_\alpha}(t_1, \dots, t_k)\}$  is fixed. By 5.1.10 it follows that  $r(A, F) \leq k$  in some subregion  $G_n \subset D_n$ . So we have obtained a contradiction to the assumption that  $r(A, F) > k$  in any subregion  $G_n \subset D_n$  and this proves the theorem.

#### REFERENCES

- [1] HILBERT, D. Mathematische Probleme. *Nachr. Akad. Wiss. Göttingen* (1900), 253-297; *Gesammelte Abhandlungen*, Bd. 3 (1935), 290-329.
- [2] OSTROWSKI, A. Über Dirichletsche Reihen und algebraische Differentialgleichungen. *Math. Z.* 8 (1920), 241-298.
- [3] HILBERT, D. Über die Gleichung neunten Grades. *Math. Ann.* 97 (1927), 243-250; *Gesammelte Abhandlungen*, Bd. 2 (1933), 393-400.
- [4] VITUSHKIN, A. G. On Hilbert's thirteenth problem. *Dokl. Akad. Nauk SSSR* 95 (1954), 701-704.
- [5] BIEBERBACH, L. Bemerkung zum dreizehnten Hilbertschen Problem. *J. Reine Angew. Math.* 165 (1931), 89-92.
- [6] ——— Einfluss von Hilberts Pariser Vortrag über „Mathematische Probleme“. *Naturwissenschaften* 51 (1930), 1101-1111.
- [7] KOLMOGOROV, A. N. On the representation of continuous functions of several variables by superpositions of continuous functions of fewer variables. *Dokl. Akad. Nauk SSSR* 108 (1956), 179-182. *Amer. Math. Soc. Transl.* (2) 17 (1961), 369-373.
- [8] ARNOL'D, V. I. On functions of three variables. *Dokl. Akad. Nauk SSSR* 114 (1957), 679-681.
- [9] KOLMOGOROV, A. N. On the representation of continuous functions of several variables by superpositions of continuous functions of one variable and addition. *Dokl. Akad. Nauk SSSR* 114 (1957), 953-956. *Amer. Math. Soc. Transl.* (2) 28 (1963), 55-59.

- [10] VITUSHKIN, A. G. and G. M. HENKIN. Linear superpositions of functions. *Uspekhi Mat. Nauk* 22, N 1 (1967), 77-124.
- [11] VITUSHKIN, A. G. On 13-th Problem of Hilbert. *Hilbert's problem*, Izdat. Nauka, 1969.
- [12] LORENTZ, G. G. On the 13-th problem of Hilbert. "Mathematical Developments Arising from the Hilbert Problems", Proceedings of Symposia in Pure Mathematics, Vol. 28 (De Kalb, Illinois, May 1974), American Mathematical Society Providence, RI, 1975.
- [13] WIMAN, A. Über die Anwendung der Tschirnhausen-Transformation auf die Reduktion algebraischer Gleichungen. *Nova Acta Soc. Sci. Upsal.* (1928), 3-8.
- [14] CHEBOTAREV, G. N. On the problem of the resolvent. *Kazan. Gos. Univ. Uch. Zap.* 114 (1954), 189-193.
- [15] — On certain questions of the problem of resolvents. *Sobranie sochinenii* (collected works), Vol. 1 (1949), 255-340.
- [16] MOROZOV, V. V. On certain questions of the problem of resolvents. *Kazan. Gos. Univ. Uch. Zap.* 114 (1954), 173-187.
- [17] LIN, V. Superpositions of algebraic functions. *Functional analysis and its applications* 10, N 1 (1976), 37-45.
- [18] ARNOL'D, V. I. Topological invariants of algebraic functions, *Functional analysis and its applications* 4, N 2 (1970), 1-9.
- [19] HOVANSKI, A. G. On superpositions of holomorphic functions with radicals. *Uspekhi Mat. Nauk* 26, N 2 (1971).
- [20] — On a representation of functions by quadrature. *Uspekhi Mat. Nauk* 26, N 4 (1971), 251-252.
- [21] VITUSHKIN, A. G. *O mnogomernykh variatsiyakh* (On multidimensional variations), Gostehizdat, Moscow 1955.
- [22] IVANOV, L. D. *Variatsii mnogestv i funktsii* (Variations of sets and functions). Izdat. Nauka, Moscow 1975.
- [23] KOLMOGOROV, A. N. Estimates of the minimal number of elements of  $\varepsilon$ -nets in various functional classes and their application of representability of functions of several variables by superpositions of functions of fewer variables. *Uspekhi Mat. Nauk* 10, N 1 (1955), 192-193; *Dokl. Akad. Nauk SSSR* 101 (1955), 192-194.
- [24] LORENTZ, G. G. Lower bounds for the degree of approximation. *Trans. Amer. Math. Soc.* 97, N 1 (1960), 25-34.
- [25] SHAPIRO, H. S. Some negative theorems of approximation theory. *Michigan Math.* 11, N 3 (1964), 211-217.
- [26] EROKHIN, V. D. On the test approximation of analytic functions by rational functions with free poles. *Dokl. Akad. Nauk SSSR* 128 (1959), 29-32.
- [27] TIHOMIROV, V. M. Diameters of sets in functional spaces and the theory of test approximations. *Uspekhi Mat. Nauk* 15, N 3 (1960), 81-120. *Russian Math. Surveys* 15, N 3 (1960), 75-111.
- [28] DUNFORD, N. and J. T. SCHWARTZ. *Linear Operators I, General Theory*. Interscience Publishers, New York and London, 1958. Translation: *Lineinye operatory, Obshchaya teoriya*, Izdat. Inost. Lit. Moscow 1962, 526-527.
- [29] KRONROD, A. S. On functions of two variables. *Uspekhi Mat. Nauk* 5, N 1 (1950), 24-134.
- [30] MENGER, K. *Kurventheorie*. Berlin-Leipzig, 1932, Chap. X.
- [31] ARNOL'D, V. I. On the representation of continuous functions of three variables by superpositions of continuous functions of two variables. *Math. Sb.* 48 (90) (1959), 3-74; 56 (98) (1962), 392. *Amer. Math. Soc. Transl.* (2) 28 (1963), 61-147.

- [32] BARI, N. Mémoire sur la représentation finie des fonctions continues. *Math. Ann.* 103 (1930), 145-248, 598-653.
- [33] ARNOL'D, V. I. On the representability of functions of several variables in the form of superpositions of functions of fewer variables. *Matem. Prosv.* 3 (1958).
- [34] LORENTZ, G. G. Metric entropy, widths and superpositions of functions. *Amer. Math. Monthly* 69 (1962), 469-485.
- [35] TIHOMIROV, V. M. Kolmogorov's work on the  $\varepsilon$ -entropy of functional classes and superpositions of functions. *Uspehi Mat. Nauk* 18, N 5 (1963), 55-92. *Russian Mathematical Surveys* 18, N 5 (1963), 51-87.
- [36] KAHANE, J.-P. Sur le théorème de superposition de Kolmogorov. *Journal of Approximation theory* 13, N 3 (1975), 229-234.
- [37] FRIDMAN, B. Improvement in the smoothness of functions in the Kolmogorov superposition theorem. *Dokl. Akad. Nauk SSSR* 177 (1967), 5.
- [38] DOSS, R. On the representation of the continuous of two variables by means of addition and continuous functions of one variable. *Colloq. Math.* 10 (1963), 249-259.
- [39] BASSALYGO, L. A. On the representation of continuous functions of two variables by means of continuous functions of one variable. *Vestnik Moskov. Univ. Ser. I, Mat. Meh.* 21 (1966), 58-63.
- [40] SPRECHER, D. A. On the structure of continuous functions of several variables. *Trans. Amer. Math. Soc.* 115 (1965), 340-355.
- [41] ARNOL'D, V. I. On the representability of functions of two variables in the form  $\kappa(\varphi(x) + \psi(y))$ . *Uspehi Mat. Nauk* 12, N 2 (1957), 119-121.
- [42] OFMAN, Yu. P. On the best approximation of functions of two variables by functions of the form  $\varphi(x) + \psi(y)$ . *Izv. Akad. Nauk SSSR Ser. Mat.* 25 (1961), 239-252.
- [43] MOTORNYI, V. P. On the question of the best approximation of functions of two variables by functions of the form  $\varphi(x) + \psi(y)$ . *Izv. Akad. Nauk SSSR Ser. Mat.* 27 (1963), 1211-1214.
- [44] SPRECHER, D. A. On the structure of representations of continuous functions of several variables as finite sums of continuous functions of one variable. *Proc. Amer. Math. Soc.* 17 (1966), 98-105.
- [45] HENKIN, G. M. On linear superpositions of continuously differentiable functions. *Dokl. Akad. Nauk SSSR* 157 (1964), 288-290. *Soviet Math. Dokl.* 5 (1964), 948-950.
- [46] FRIDMAN, B. The set of linear superpositions is nowhere dense in the space of continuous functions of several variables. *Izv. Akad. Nauk SSSR Ser. Mat.* 36, N 4 (1972), 814-846.
- [47] VITUSHKIN, A. G. Some properties of linear superpositions of smooth functions. *Dokl. Akad. Nauk SSSR* 156 (1964), 1003-1006. *Soviet Math. Dokl.* 5 (1964), 741-744.
- [48] ——— Proof of the existence of analytic functions of several variables not representable by linear superpositions of continuously differentiable functions of fewer variables. *Dokl. Akad. Nauk SSSR* 156 (1964), 1258-1261. *Soviet Math. Dokl.* 5 (1964), 793-796.
- [49] ——— *Otsenka slozhnosti zadachi tabulirovaniya* (Estimation of the complexity of the tabulation problem). Fizmatgiz, Moscow 1959. *Theory of the transmission and processing of information*. Pergamon press, 1961.
- [50] KOLMOGOROV, A. N. and V. M. TIHOMIROV.  $\varepsilon$ -entropy and  $\varepsilon$ -capacity of sets in function spaces. *Uspehi Mat. Nauk* 14, N 2 (1959), 3-86.
- [51] BUSLAEV, V. I. and A. G. VITUSHKIN. An estimate of the code length of signals with finite spectrum in connection with sound recording problems. *Izv. Akad. Nauk SSSR Ser. Mat.* 38, N 4 (1974), 867-895.

- [52] VITUSHKIN, A. G. Coding of signals with finite spectrum and sound recording problems. *Proceedings of the International Congress of Mathematicians*, Vancouver, 1974.
- [53] SMUSHKO, V. V. Entropy of the class of entire functions with the frequency dependent metric. *Izv. Akad. Nauk SSSR Ser. Mat.* 40, N 5 (1976), 1173-1186.
- [54] HEDBERG, T. Sur les réarrangements de fonctions de la classe  $A$  et les ensembles d'interpolation pour  $A(D^2)$ . *C. R. Acad. Sc. Paris* 270 (1970), 1491-1494.

(Reçu le 28 avril 1977)

A. G. Vitushkin

Steklov Mathematical Institute  
Ul. Vavilova, 42  
Moscow 117333, USSR