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Chapter 4. — Linear superpositions

In this chapter we prove that there exist analytic functions which are not representable by means of linear superpositions of smooth functions of one variable.

§ 1. *Notation*

Throughout we assume that all the functions are defined and continuous for all values of the arguments. If we say that a function is continuously differentiable, we mean by this that its first partial derivatives are defined and continuous for all values of the arguments; $z = (x, y)$ is the point of the plane with coordinates x and y; grad $[q(z)]$ is the gradient of the function $q(z)$, that is, the vector-function with coordinates $\frac{\partial q}{\partial x}$ and $\frac{\partial q}{\partial y}$;

$$D\left(\frac{q_1, q_2}{x, y}\right) = \begin{vmatrix} \frac{\partial q_1}{\partial x} & \frac{\partial q_1}{\partial y} \\ \frac{\partial q_2}{\partial x} & \frac{\partial q_2}{\partial y} \end{vmatrix}$$

is the Jacobian of the pair of functions q_1 and q_2.

$q(D)$ is the image of the set D under the mapping effected by the function $q(x, y)$; $q^{-1}(\delta)$ is the complete inverse image of the interval δ on the axis of values of the function $q(x, y)$.

$e(q, t)$ is the set of level t of the function $q = q(x, y)$.

$\tau(e, z)$ is the unit tangent vector to the curve e at the point $z \in e$.

$\gamma(\tau_1, \tau_2)$ is the absolute value of the acute angle between the vectors τ_1 and τ_2.

$h_1(e)$ is the length of the set e.

$d_1(e)$ is the one-dimensional diameter of the set e.

$0(\gamma)$ is a quantity bounded by a constant depending only on γ.

$\rho(A_1, A_2)$ is the distance between the sets A_1 and A_2 in the sense of deviation, more precisely

$$\rho(A_1, A_2) = \max\{\sup_{z_1 \in A_1} \inf_{z_2 \in A_2} \rho(z_1, z_2), \sup_{z_2 \in A_2} \inf_{z_1 \in A_1} \rho(z_1, z_2)\},$$

where $\rho(z_1, z_2)$ is the distance between the points z_1 and z_2.